Single-Degree-of-Freedom (SDOF) and Response Spectrum

Ahmed Elgamal

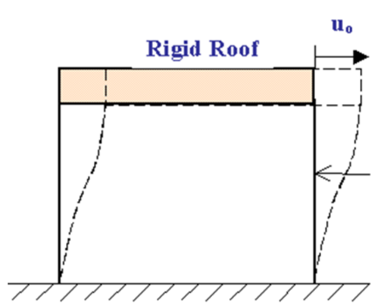
Dynamics of a Simple Structure

The Single-Degree-Of-Freedom (SDOF) Equation

References

Dynamics of Structures, Anil K. Chopra, Prentice Hall, New Jersey, ISBN 0-13-855214-2 (Chapter 3).

Elements of Earthquake Engineering And Structural Dynamics, André Filiatrault, Polytechnic International Press, Montréal, Canada, ISBN 2-553-00629-4 (Section 4.2.3).



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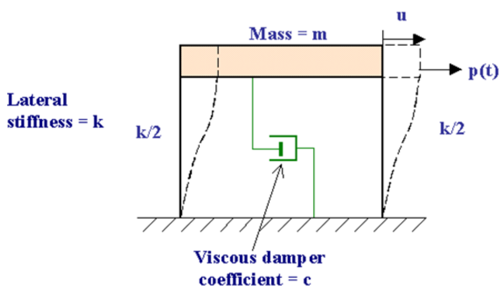
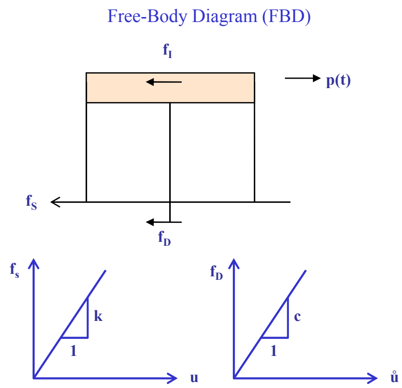
Equation of motion (external force)



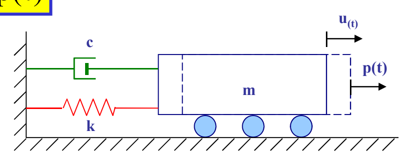


$$f_I + f_D + f_s = p(t)$$

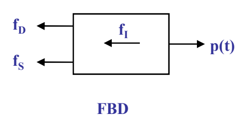
where f_I, f_D, and f_s are forces due to Inertia, Damping and Stiffness respectively:

$$f_s = k\,u \qquad f_D = c\,\dot{u}$$

$$m\ddot{u} + c\dot{u} + ku = p(t)$$



Mass-spring damper system



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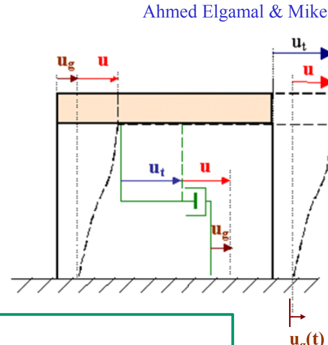
Earthquake Ground Motion (u_g)

$$f_I + f_D + f_s = 0$$

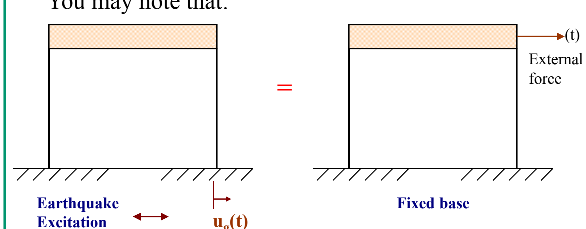
$$f_I = m\ddot{u}_t = m(\ddot{u} + \ddot{u}_g)$$

$$m(\ddot{u} + \ddot{u}_g) + c\dot{u} + ku = 0$$

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$$



You may note that:



u = relative displacement (displacement of the structure relative to the ground)
u_t = total displacement
f_I = m x (total or absolute acceleration)

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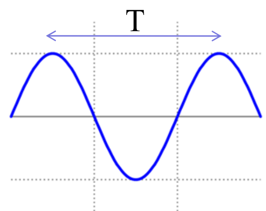
Undamped natural frequency

Property of structure when allowed to vibrate freely without external excitation

$\omega = \sqrt{\frac{k}{m}}$ Undamped natural circular frequency of vibration (radians/second)

$f = \frac{\omega}{2\pi}$ natural cyclic frequency of vibration (cycles/second or 1/second or Hz)

$T = \frac{1}{f}$ natural period of vibration (second)



T is the time required for one cycle of free vibration

If damping is present, replace ω by ω_D

where $\omega_D = \omega\sqrt{1-\xi^2}$ natural frequency*, and

$\xi = \frac{c}{2m\omega}$ fraction of critical damping coefficient (**damping ratio**, zeta)

$= \frac{c}{c_c}$ $= \frac{c}{2\sqrt{km}}$ (dimensionless measure of damping)

$c_c = 2m\omega = 2\sqrt{km}$ c_c = critical damping

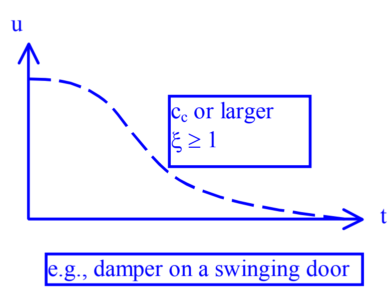
* Note: In earthquake engineering, $\omega_D = \omega$ approximately, since ζ is usually below 0.2 (or 20%)

c_c is the least level of damping that prevents oscillation

In general $\xi < 0.2$, i.e., $\omega_D \approx \omega$, $f_D \approx f$, $T = T_D$

ξ may be in the range of 0.02 – 0.2 or 2% - 20%

5% is sometimes a typical value.



in terms of ξ

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$$

$$\ddot{u} + \frac{c}{m}\dot{u} + \frac{k}{m}u = -\ddot{u}_g$$

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g$$

Note: After the phase of forced vibration (due to external force or base excitation, or initial conditions), the structure continues to vibrate in a "free vibration" mode till it stops due to damping. The ratio between amplitude in two successive cycles is

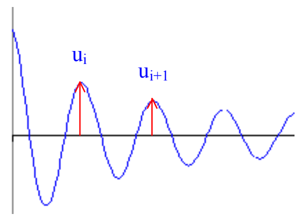
$$\frac{u_i}{u_{i+1}} \approx e^{2\pi\xi}$$

where we define the logarithmic decrement as

$$\delta = 2\pi\xi = \ln\left(\frac{u_i}{u_{i+1}}\right)$$ if you measure a free vibration response you can find ξ.

Note: for peaks j cycles apart

$$\ln\left(\frac{u_i}{u_{i+j}}\right) = j\delta = 2j\pi\xi$$



Free vibration

Why is $c_c = 2m\omega = 2\sqrt{km}$

Critical viscous damping

The free vibration equation may be written as

$$m\ddot{x} + c\dot{x} + kx = 0$$

and the general solution is

$$x = C_1 e^{\left[\frac{-c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)}\right] t} + C_2 e^{\left[\frac{-c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)}\right] t}$$

if $\left(\frac{c}{2m}\right)^2 = \frac{k}{m}$, the radical part of the exponent will vanish. This will produce aperiodic response (non-oscillatory). In this case

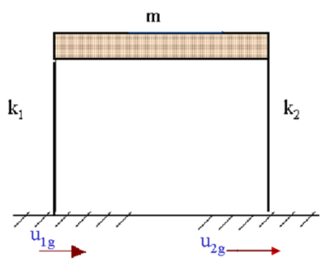
$$\frac{c^2}{(2m)^2} = \frac{k}{m}$$ or $c = 2\sqrt{km} = c_c$

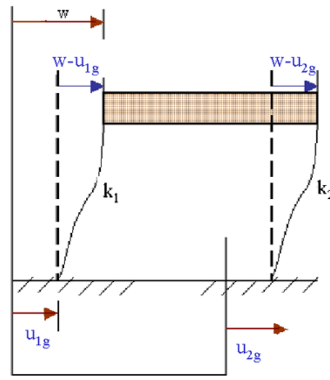
since $\omega = \sqrt{\frac{k}{m}}$, c_c is also equal to $2m\omega$ (note that $2\sqrt{km} = 2\sqrt{m\omega^2 m} = 2m\omega$)

and also $c_c = 2\sqrt{km} = 2\sqrt{k(k/\omega^2)} = \frac{2k}{\omega}$

Multi-support excitation

w = absolute displacement, u_g = ground displacement





$$m\ddot{w} + k_1(w - u_{1g}) + k_2(w - u_{2g}) = 0$$

$$m\ddot{w} + (k_1 + k_2)w = k_1 u_{1g} + k_2 u_{2g}$$

if $u_{1g} = u_{2g} = u_g$

$$\frac{c}{c_c} = \xi \quad \text{(0.05 for example)}$$

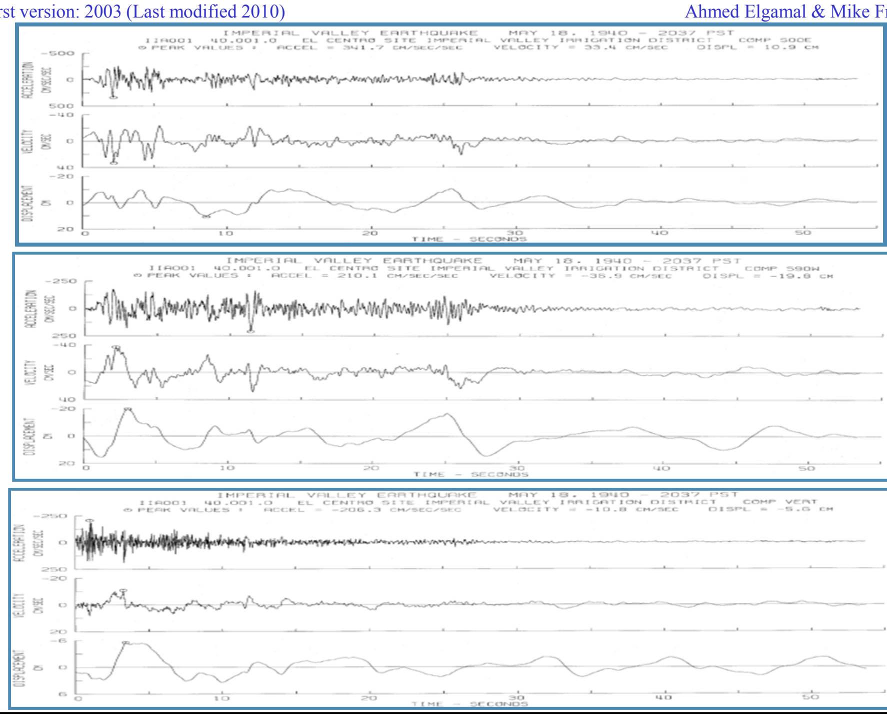
$$m\ddot{w} + (k_1 + k_2)w = (k_1 + k_2)u_g$$

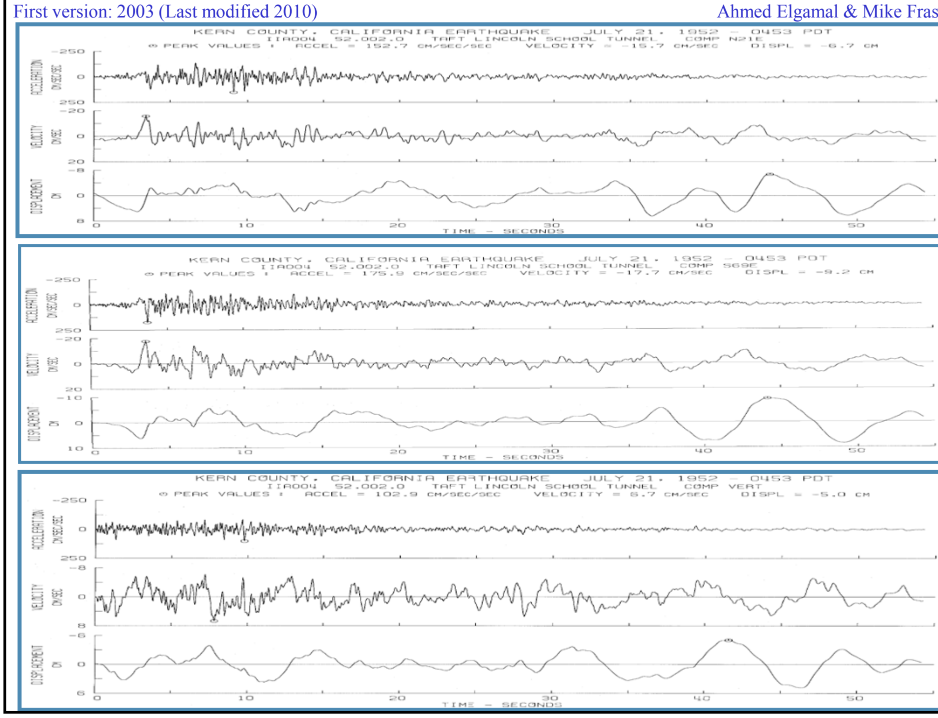
but $w = u + u_g$

$$m\ddot{u} + (k_1 + k_2)u_g + (k_1 + k_2)u = -m\ddot{u}_g + (k_1 + k_2)u_g$$

$$c_c = 2\sqrt{(k_1 + k_2)m}$$

or $m\ddot{u} + (k_1 + k_2)u = -m\ddot{u}_g$





Deformation Response Spectrum

For a given EQ excitation calculate $|u_{max}|$ from SDOF response with a certain ξ and within a range of natural periods or frequencies.

$|u_{max}|$ for each frequency will be found from the computed u(t) history at this frequency.

A plot of $|u_{max}|$ vs. natural period is constructed representing the deformation (or displacement) response spectrum (S_d).

From this figure, one can directly read the maximum relative displacement of any structure of natural period T (and a particular value of ξ as damping)

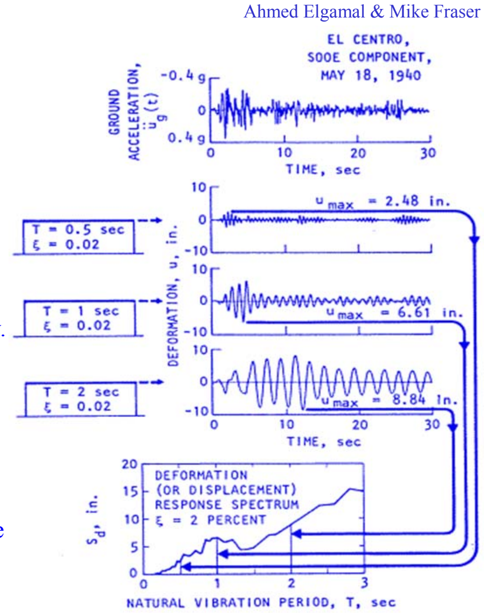


Figure 17. Computation of deformation (or displacement) response spectrum

From: Chopra, Dynamics of Structures, A Primer

Example

Units for natural frequency calculation (SI units)

Weight = 9.81 kN = 9810 N = 9810 kg (m /s^2)

Gravity (g) = 9.81m/s^2

Mass (m) = W/g = 9180/9.81 = 1000 kg

Stiffness (k) = 81 kN/m = 81000 N/m = 81000 kg (m/s^2)/m = kg /s^2

ω = SQRT (k/m) = SQRT (81000/1000) = 9 radians/sec

f = ω / (2 π) = 1.43 Hz (units of 1/s or cycles/sec)

Units for natural frequency calculation (English units)

Weight (W) = 193.2 Tons = 193.2 (2000) = 386,400 lbs = 386.4 kips

g = 386.4 in/sec/sec (or in/s^2)

Mass (m) = W/g = 1.0 kips s^2 /in

Stiffness (k) = 144 kips/in

ω =SQRT (k/m) = 12 radians/sec

f = ω / (2 π) = 1.91 Hz (units of 1/s or cycles/sec)

Note: The weight of an object is the force of gravity on the object and may be defined as the mass times the acceleration of gravity, w = mg. Weight is what is measured by a scale (e.g., weight of a person). Since the weight is a force, its SI unit is Newton.

Concept of Equivalent lateral force f_s

If f_s is applied as a static force, it would cause the deformation u.

Thus at any instant of time:

$f_s = ku(t)$, or in terms of the mass $f_s(t) = m\omega^2 u(t)$

$$\omega = \sqrt{\frac{k}{m}}$$

The maximum force will be

$$f_{s,max} = m\omega^2 u_{max} = ku_{max} = mS_a = kS_d$$

$$S_a = \frac{k}{m} S_d \qquad S_a = \omega^2 S_d$$

S_d = deformation or displacement response spectrum

$S_a = \omega^2 S_d$ = pseudo-acceleration response spectrum

The maximum strain energy E_{max} stored in the structure during shaking is:

$$E_{max} = \frac{1}{2} ku^2_{max} = \frac{1}{2} kS_d^2 = \frac{1}{2}\frac{k}{\omega^2}\omega^2 S_d^2 = \frac{1}{2} m\omega^2 S_d^2 = \frac{1}{2} mS_v^2$$

where $S_v = \omega S_d$ = pseudo-velocity response spectrum

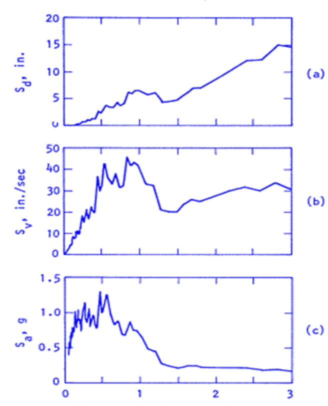


Figure 19. (a) Deformation (or Displacement), (b) pseudo-velocity and (c) pseudoacceleration response spectra. El Centro ground motion—S00°E component. Damping ratio ξ = 2 percent

From: Chopra, Dynamics of Structures, A Primer

Note that S_d, S_v and S_a are inter-related by

$S_a = \omega^2 S_d = \omega S_v$

S_a, S_v are directly related to S_d
by ω^2 and ω respectively or by $(2\pi f)^2$ and $2\pi f$;
or $\left(\frac{2\pi}{T}\right)^2$ and $\left(\frac{2\pi}{T}\right)$ as shown in Figure.

Due to this direct relation, a 4-way plot
is usually used to display S_a, S_v and S_d
on a single graph as shown in Figure.
In this figure, the logarithm of
period T, S_a, S_v and S_d is used.

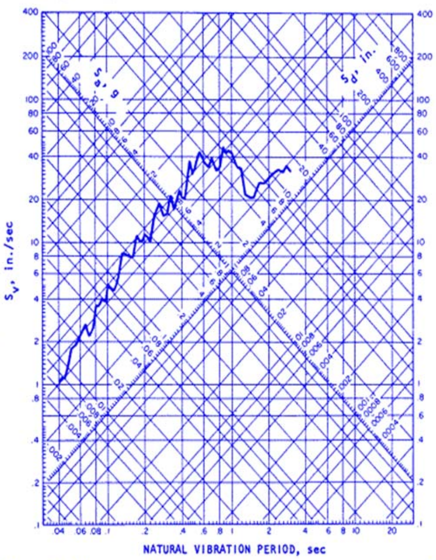


Figure 20. Four-way logarithmic plot of response spectrum. El Centro ground motion—S00°E component. Damping ratio ξ = 2 percent

From: Chopra, Dynamics of Structures, A Primer

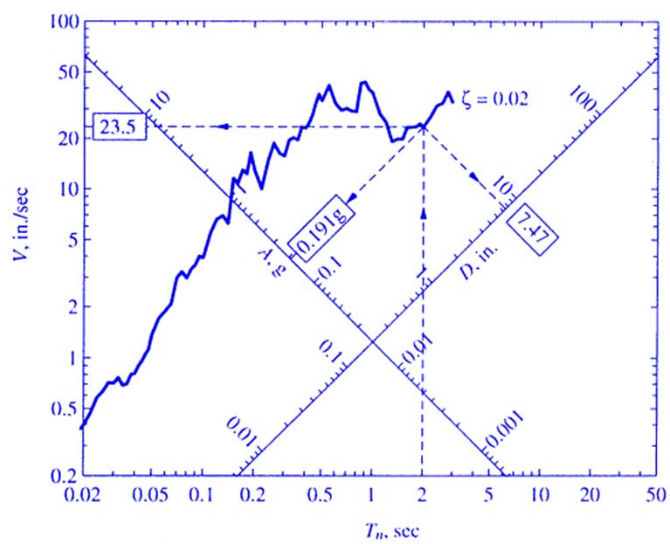


Figure 6.6.3 Combined *D–V–A* response spectrum for El Centro ground motion; $\zeta = 2\%$.

From: Chopra, Dynamics of Structures

In order to cover the damping range of interest, it is common to perform the same calculations for ξ =0.0, 0.02, 0.05, 0.10, and 0.20 (see Figure)

Typical Notation:

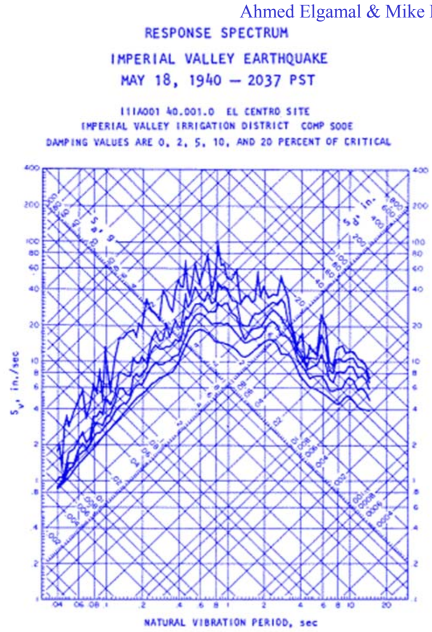
$S_v \equiv PSV \equiv V$

$S_a \equiv PSA \equiv A$

$S_d \equiv \; SD \equiv D$

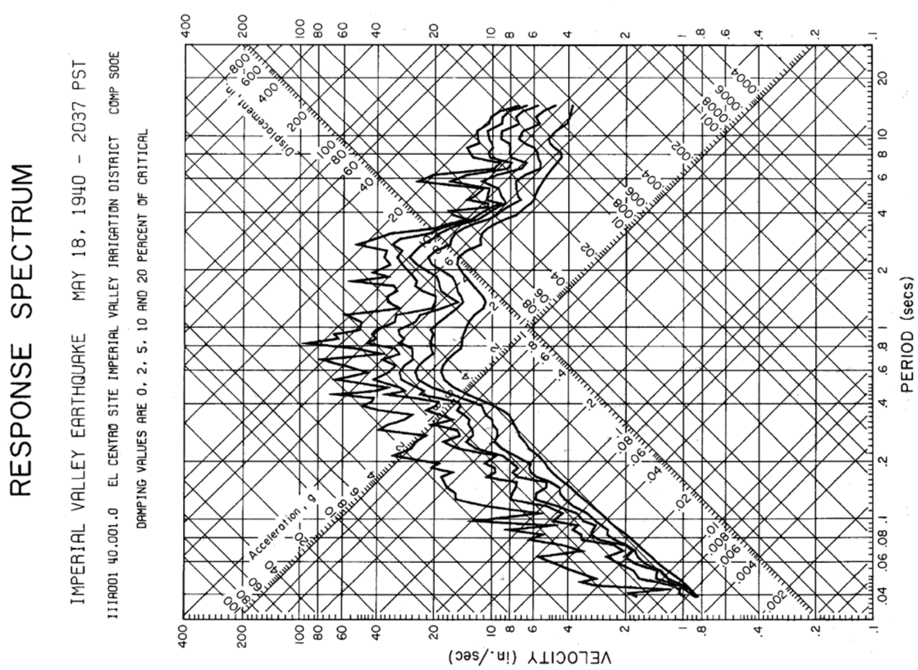
Example (El-Centro motion):
Find maximum displacement and base shear of tower with f = 2 Hz, ξ = 2% and k = 1.5 MN/m

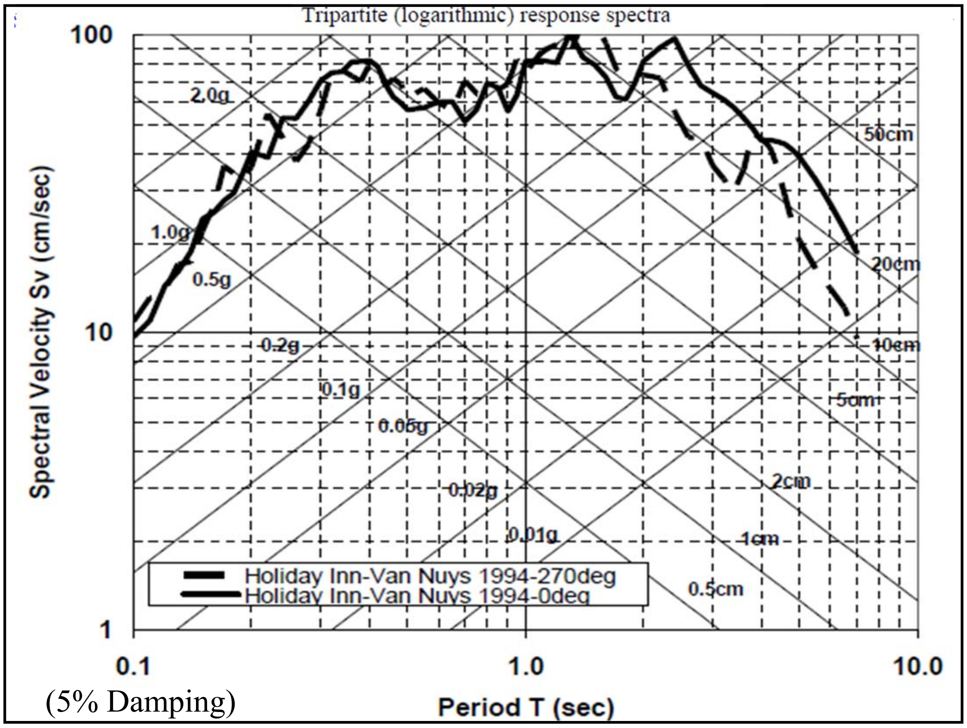
Period T = 1/f = 0.5 second
S_d = 2.5 inches = 0.0635 m
$Force_{max} = ku_{max}$
= 1.5 MN/m x 0.0635m = 95.25 kN



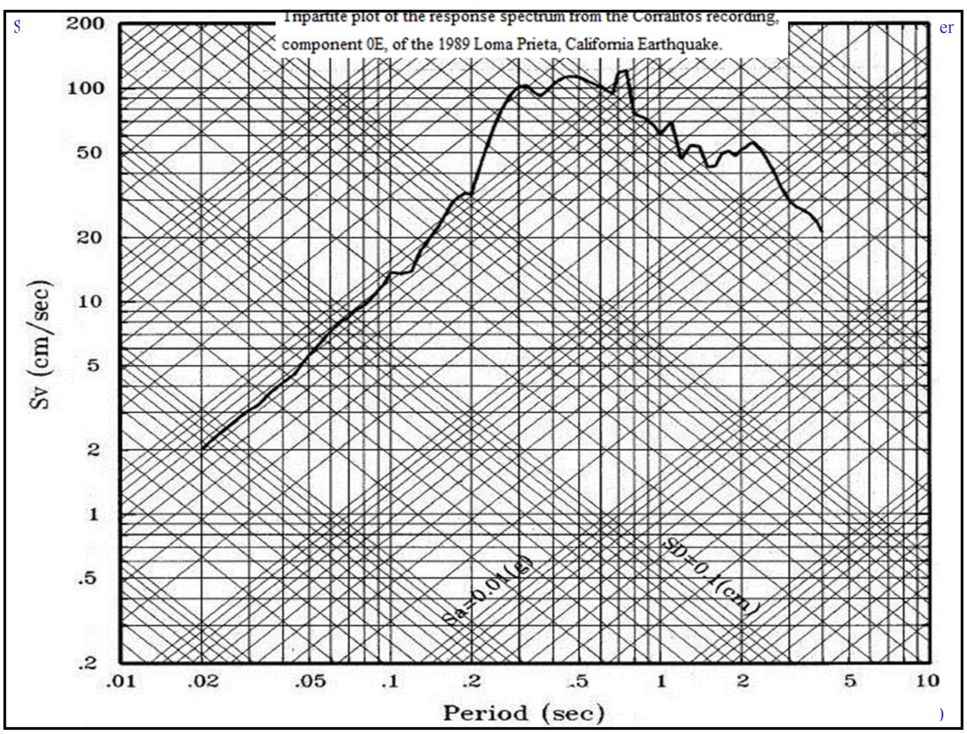
From: Chopra, Dynamics of Structures, A Primer

Figure 21. Four-way logarithmic plot of response spectrum. El Centro ground motion—S00°E component (after Hudson, 1979)





Tripartite (logarithmic) response spectra
Spectral Velocity Sv (cm/sec)
Period T (sec)
(5% Damping)
Holiday Inn-Van Nuys 1994-270deg
Holiday Inn-Van Nuys 1994-0deg
2.0g
1.0g
0.5g
0.2g
0.1g
0.05g
0.02g
0.01g
50cm
20cm
10cm
5cm
2cm
1cm
0.5cm
100
10
1
0.1
1.0
10.0



Tripartite plot of the response spectrum from the Corralitos recording,
component 0E, of the 1989 Loma Prieta, California Earthquake.
Sv (cm/sec)
Period (sec)
Sa=0.01(g)
SD=0.1(cm)
200
100
50
20
10
5
2
1
.5
.2
.01
.02
.05
.1
.2
.5
1
2
5
10

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Inspection of this figure shows that the maximum response at short period (high frequency stiff structure) is controlled by the ground acceleration, low frequency (long period) by ground displacement, and intermediate period by ground velocity.

Get copy of El-Centro
(May 18, 1940)
earthquake record
S00E (N-S component)
ftp nisee.ce.berkeley.edu
(128.32.43.154)
login: anonymous
password: your_indent
cd pub/a.k.chopra
get el_centro.data
quit

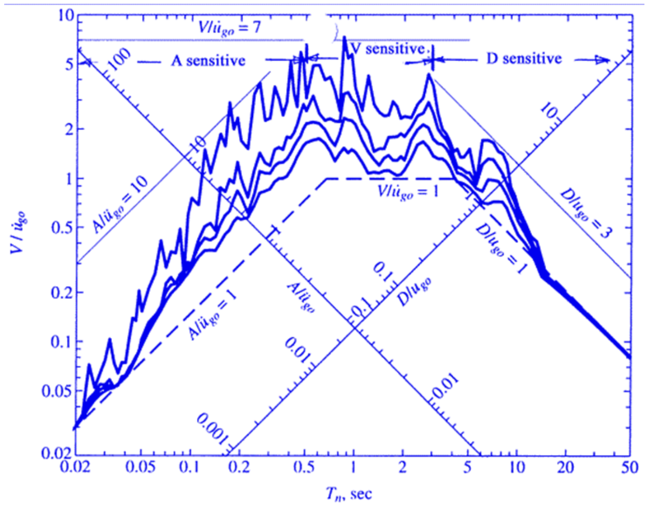

Figure 6.8.2 Response spectrum for El Centro ground motion plotted with normalized scales $A/\ddot{u}_{go}$, $V/\dot{u}_{go}$, and D/u_{go}; $\zeta = 0, 2, 5,$ and 10%.

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Note that response spectrum for relative velocity may be obtained from the SDOF response history, and similarly for $\ddot{u}^t = (\ddot{u} + \ddot{u}_g)$. These spectra are known as relative velocity and total acceleration response spectra, and are different from the pseudo velocity and pseudo acceleration spectra S_v and S_a (which are directly related to S_d).

e.g. for $\xi = 0\%$

$m(\ddot{u} + \ddot{u}_g) + ku = 0$

or $(\ddot{u} + \ddot{u}_g) + \omega^2 u = 0$

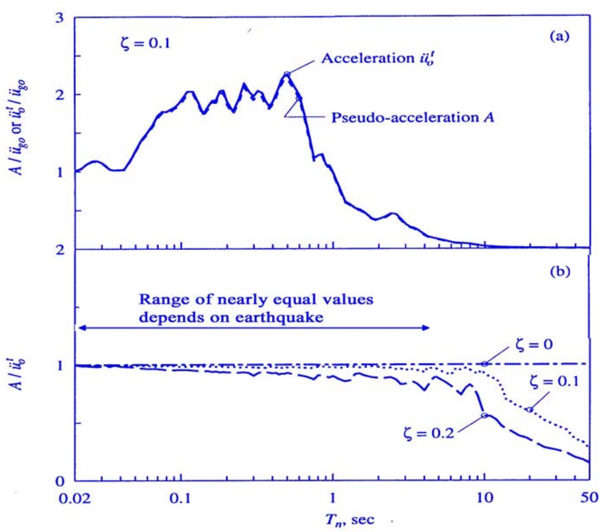

Figure 6.12.2 (a) Comparison between pseudo-acceleration and acceleration response spectra; $\zeta = 10\%$; (b) ratio $A/\ddot{u}_o^t$ for $\zeta = 0, 10,$ and 20%.

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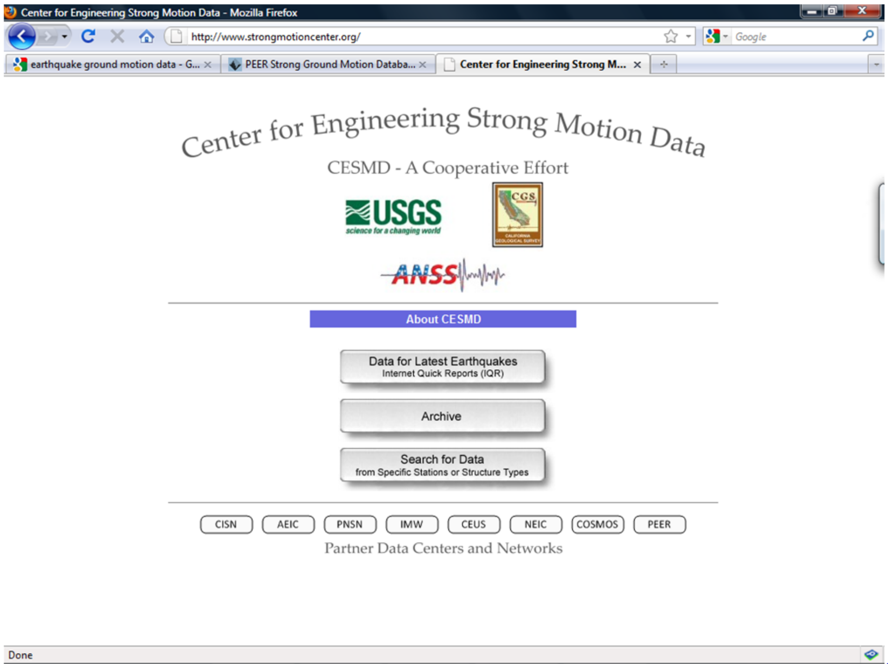
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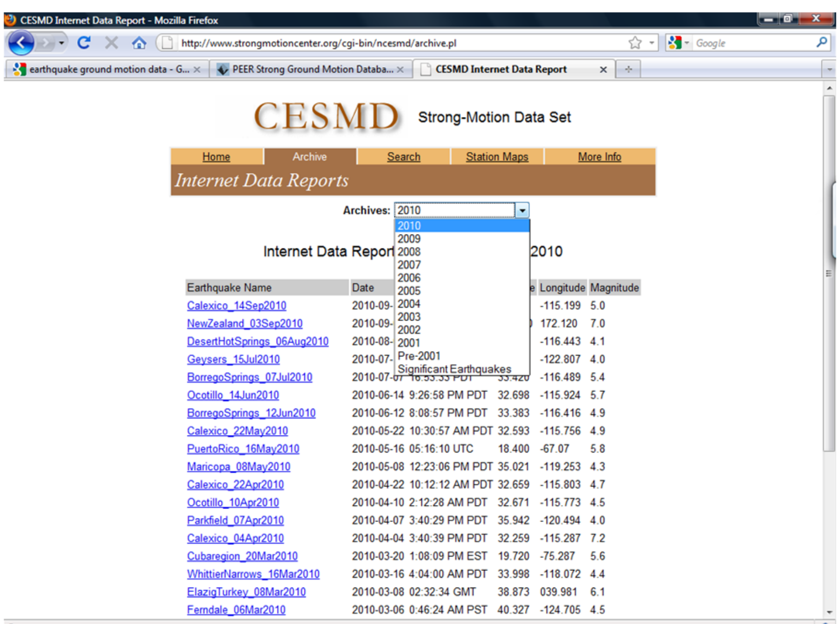
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Earthquake Name Date Longitude Magnitude
Calexico_14Sep2010 2010-09- -115.199 5.0
NewZealand_03Sep2010 2010-09- 172.120 7.0
DesertHotSprings_06Aug2010 2010-08- -116.443 4.1
Geysers_15Jul2010 2010-07- -122.807 4.0
BorregoSprings_07Jul2010 2010-07- -116.489 5.4
Ocotillo_14Jun2010 2010-06-14 9:26:58 PM PDT 32.698 -115.924 5.7
BorregoSprings_12Jun2010 2010-06-12 8:08:57 PM PDT 33.383 -116.416 4.9
Calexico_22May2010 2010-05-22 10:30:57 AM PDT 32.593 -115.756 4.9
PuertoRico_16May2010 2010-05-16 05:16:10 UTC 18.400 -67.07 5.8
Maricopa_08May2010 2010-05-08 12:23:06 PM PDT 35.021 -119.253 4.3
Calexico_22Apr2010 2010-04-22 10:12:12 AM PDT 32.659 -115.803 4.7
Ocotillo_10Apr2010 2010-04-10 2:12:28 AM PDT 32.671 -115.773 4.5
Parkfield_07Apr2010 2010-04-07 3:40:29 PM PDT 35.942 -120.494 4.0
Calexico_04Apr2010 2010-04-04 3:40:39 PM PDT 32.259 -115.287 7.2
Cubaregion_20Mar2010 2010-03-20 1:08:09 PM EST 19.720 -75.287 5.6
WhittierNarrows_16Mar2010 2010-03-16 4:04:00 AM PDT 33.998 -118.072 4.4
ElazigTurkey_08Mar2010 2010-03-08 02:32:34 GMT 38.873 039.981 6.1
Ferndale_06Mar2010 2010-03-06 0:46:24 AM PST 40.327 -124.705 4.5
Done

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Internet Data Reports

Archives: Significant Earthquakes

Significant Earthquakes

Earthquake Name	Date	Time	Latitude	Longitude	Magnitude
NewZealand_03Sep2010	2010-09-03	16:35:46 UTC	-43.530	172.120	7.0
Calexico_04Apr2010	2010-04-04	3:40:39 PM PDT	32.259	-115.287	7.2
Chile_27Feb2010	2010-02-27	06:34:14 UTC	-35.846	-72.719	8.8
HaitiRegion_12Jan2010	2010-01-12	12:21:53 UTC	18.457	-72.533	7.0
Ferndale_09Jan2010	2010-01-09	4:27:38 PM PST	40.645	-124.763	6.5
SamoaIslands_29Sep2009	2009-09-29	17:48:10 UTC	-15.509	-172.034	8.0
LAquilaItaly_06Apr2009	2009-04-06	01:32:39 UTC	42.334	13.334	5.8
Sumatra_12Sep2007	2007-09-12	11:10:26 GMT	-4.520	101.374	8.4
Sumatra_Aftershock_1_12Sep2007	2007-09-12	23:49:04 GMT	-2.506	100.906	7.9
Hawaii_15Oct2006	2006-10-15	7:07:48 AM HST	19.820	-156.027	6.7
Parkfield_28Sep2004	2004-09-28	10:15:24 AM PDT	35.815	-120.374	6.0
SanSimeon_22Dec2003	2003-12-22	11:15:56 AM PST	35.706	-121.101	6.5
HectorMine99	1999-10-16	2:46:45 AM PDT	34.600	-116.270	7.1
Northridge_17Jan1994	1994-01-17	4:30:00 AM PST	34.209	-118.541	6.4
BigBear92	1992-06-28	8:05:31 AM PDT	34.201	-116.826	6.5
Landers92	1992-06-28	4:57:31 AM PDT	34.216	-116.433	7.3
Petrolia_25Apr1992	1992-04-25	11:06:05 AM PDT	40.380	-124.230	7.1
SierraMadre91	1991-06-28	7:43:00 AM PDT	34.262	-118.002	5.8
LomaPrieta_17Oct1989	1989-10-17	5:04:00 PM PDT	37.037	-121.883	7.0
Whittier87	1987-10-01	7:42:20 AM PDT	34.067	-118.078	6.1

Done

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CESMD Internet Data Report - Mozilla Firefox

http://www.strongmotioncenter.org/cgi-bin/ncesmd/iqr_dist_DM2.pl?IQRID=Landers92&SFlag=0&Flag

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CESMD *Internet Data Report*

USGS ANSS CGS

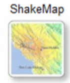
Landers Earthquake of 28 Jun 1992

7.3 ML, 04:57:31 PDT, 34.22N 116.43W Depth 1.1 km

CISN Region of ANSS

Interactive Map

ShakeMap



Report | Download Table

Last Update: 2010-08-25 16:25:16

Station	Code /ID	Network	Distance (km)		Horiz Apk (g)		View	Download
			Epic.	Fault	Ground6	Struct.		
Indio - Jackson Road	5294	USGS	55.9	42.5	**0.408**	--	- -	
San Bernardino - County Services Center	5245	USGS	99.8	79.6	**0.376**	--	- -	
Joshua Tree - Fire Station	22170	CGS	14.0	10.1	**0.290**	--		
Yermo - Fire Station	22074	CGS	84.0	23.2	**0.250**	--		
Morongo Valley - Fire Station	5071	USGS	23.0	15.3	**0.220**	--	- -	
Fun Valley - Reservoir No. 361	5069	USGS	32.1	18.5	**0.217**	--	- -	
Big Bear Lake - Civic Center Grounds	22561	CGS	46.0	45.6	**0.180**	--		
San Bernardino - I10/215 Interchange Br	23631	CGS	81.0	80.7	**0.180**	0.820		
Desert Hot Springs - Fire Station	12149	CGS	29.0	16.4	**0.180**	--		

Transferring data from www.strongmotioncenter.org...

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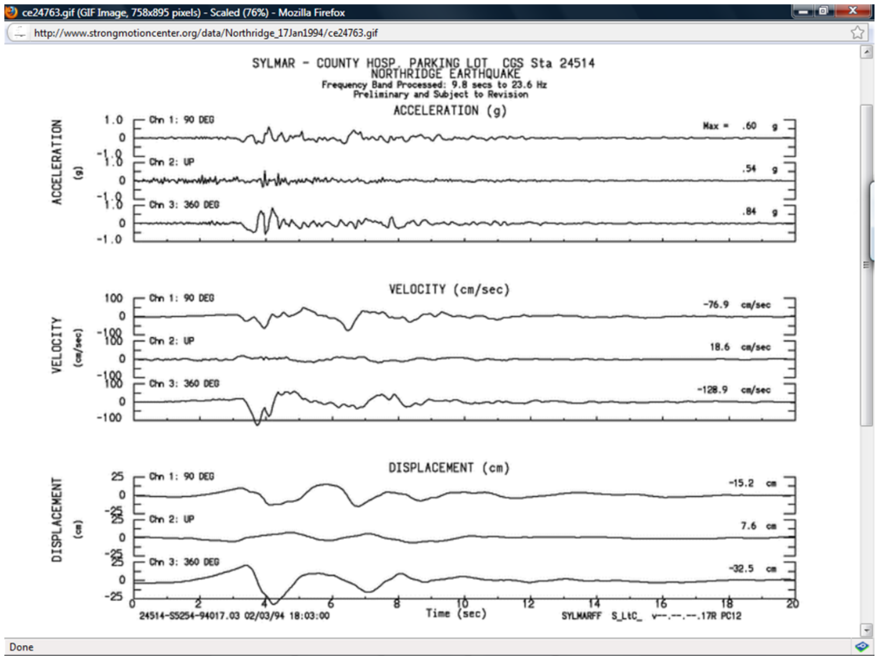

27

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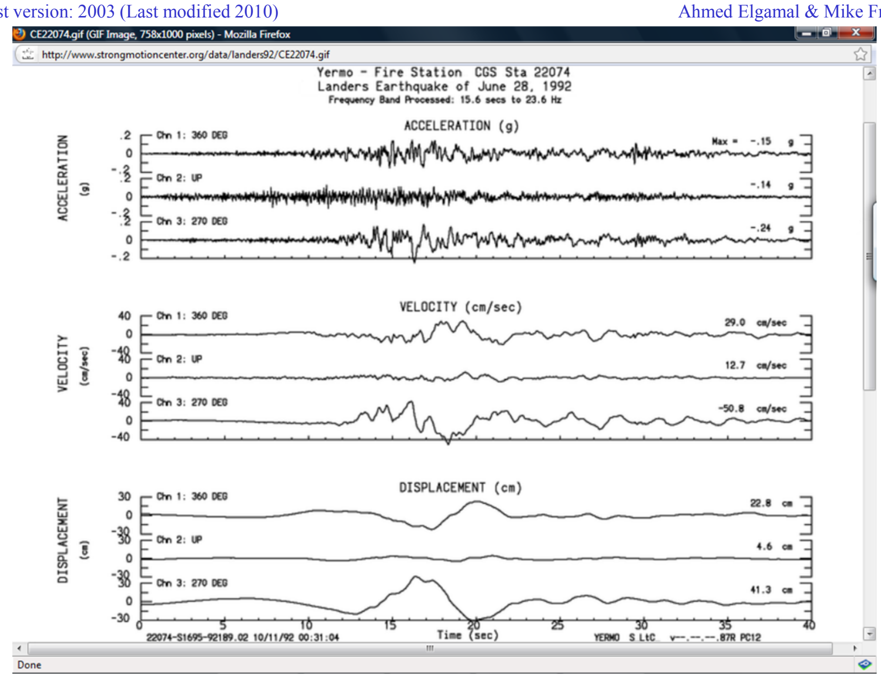
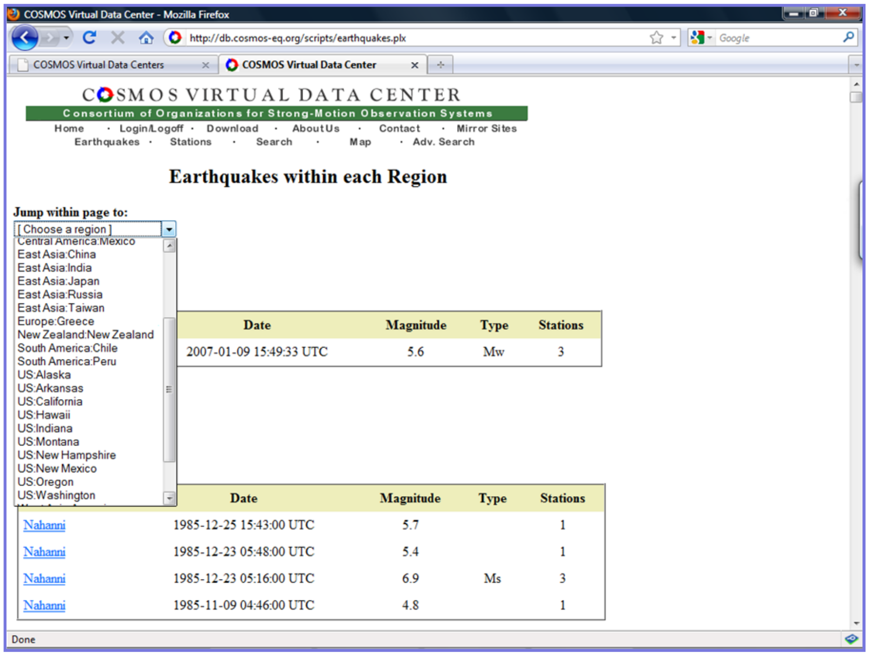


28



COSMOS Virtual Data Center - Mozilla Firefox

http://db.cosmos-eq.org/scripts/earthquakes.plx

COSMOS VIRTUAL DATA CENTER

Consortium of Organizations for Strong-Motion Observation Systems

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Earthquakes within each Region

Jump within page to:

[Choose a region]

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East Asia:China
East Asia:India
East Asia:Japan
East Asia:Russia
East Asia:Taiwan
Europe:Greece
New Zealand:New Zealand
South America:Chile
South America:Peru
US:Alaska
US:Arkansas
US:California
US:Hawaii
US:Indiana
US:Montana
US:New Hampshire
US:New Mexico
US:Oregon
US:Washington

Date	Magnitude	Type	Stations
2007-01-09 15:49:33 UTC	5.6	Mw	3

	Date	Magnitude	Type	Stations
Nahanni	1985-12-25 15:43:00 UTC	5.7		1
Nahanni	1985-12-23 05:48:00 UTC	5.4		1
Nahanni	1985-12-23 05:16:00 UTC	6.9	Ms	3
Nahanni	1985-11-09 04:46:00 UTC	4.8		1

Done

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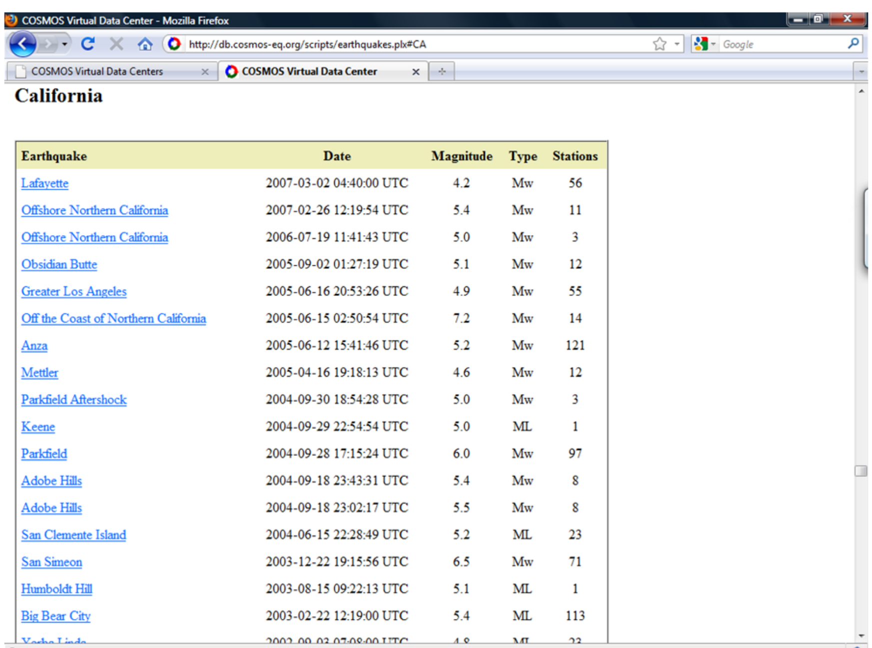

COSMOS Virtual Data Center - Mozilla Firefox

http://db.cosmos-eq.org/scripts/earthquakes.plx#CA

California

Earthquake	Date	Magnitude	Type	Stations
Lafayette	2007-03-02 04:40:00 UTC	4.2	Mw	56
Offshore Northern California	2007-02-26 12:19:54 UTC	5.4	Mw	11
Offshore Northern California	2006-07-19 11:41:43 UTC	5.0	Mw	3
Obsidian Butte	2005-09-02 01:27:19 UTC	5.1	Mw	12
Greater Los Angeles	2005-06-16 20:53:26 UTC	4.9	Mw	55
Off the Coast of Northern California	2005-06-15 02:50:54 UTC	7.2	Mw	14
Anza	2005-06-12 15:41:46 UTC	5.2	Mw	121
Mettler	2005-04-16 19:18:13 UTC	4.6	Mw	12
Parkfield Aftershock	2004-09-30 18:54:28 UTC	5.0	Mw	3
Keene	2004-09-29 22:54:54 UTC	5.0	ML	1
Parkfield	2004-09-28 17:15:24 UTC	6.0	Mw	97
Adobe Hills	2004-09-18 23:43:31 UTC	5.4	Mw	8
Adobe Hills	2004-09-18 23:02:17 UTC	5.5	Mw	8
San Clemente Island	2004-06-15 22:28:49 UTC	5.2	ML	23
San Simeon	2003-12-22 19:15:56 UTC	6.5	Mw	71
Humboldt Hill	2003-08-15 09:22:13 UTC	5.1	ML	1
Big Bear City	2003-02-22 12:19:00 UTC	5.4	ML	113

Done

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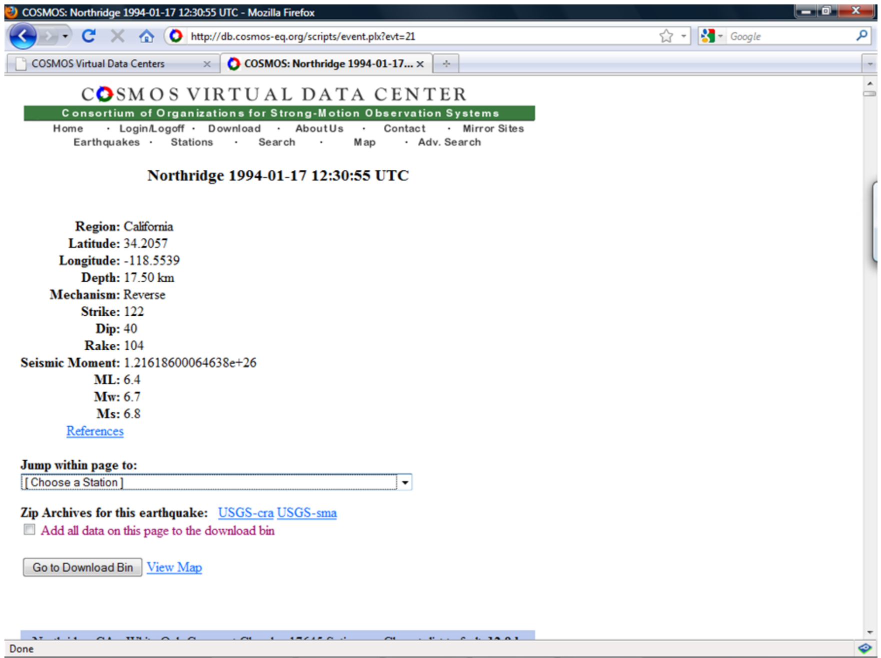

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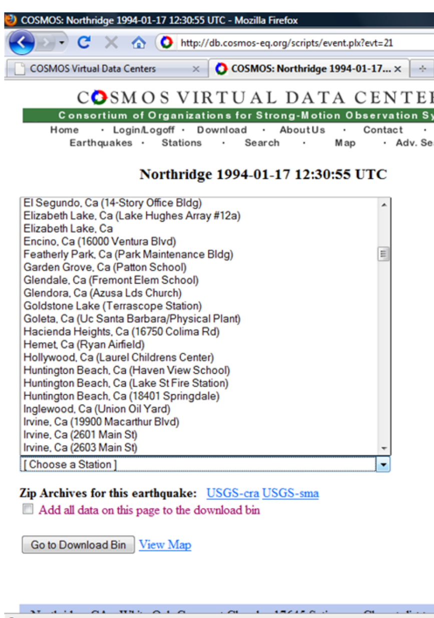

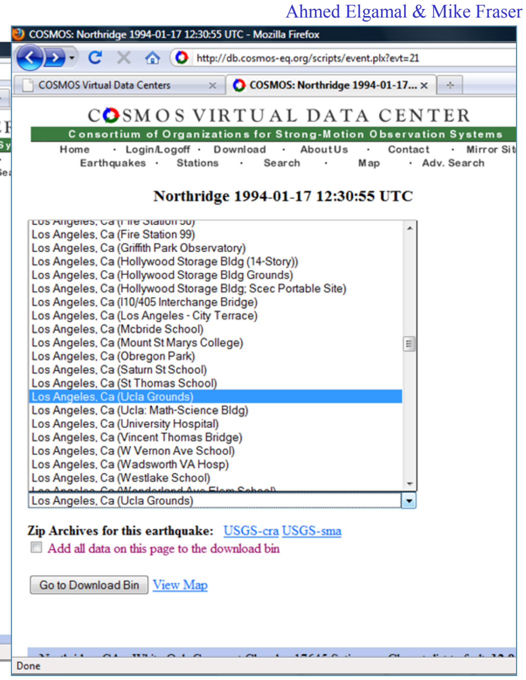

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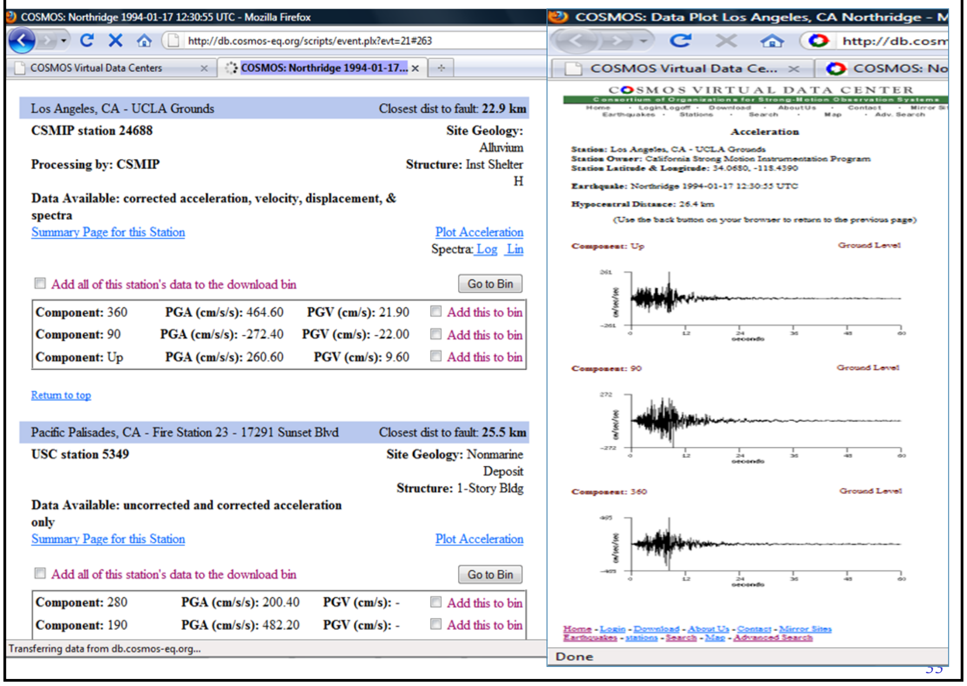

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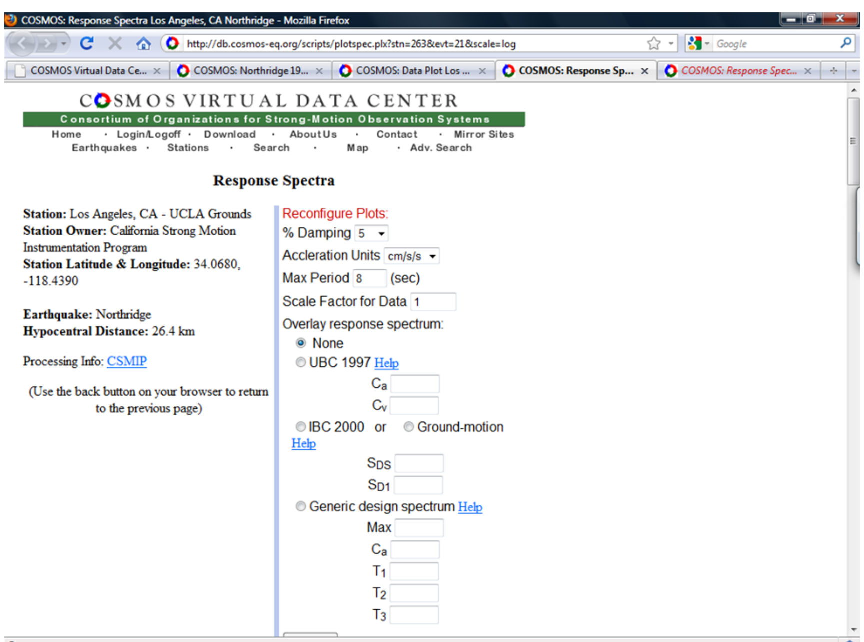

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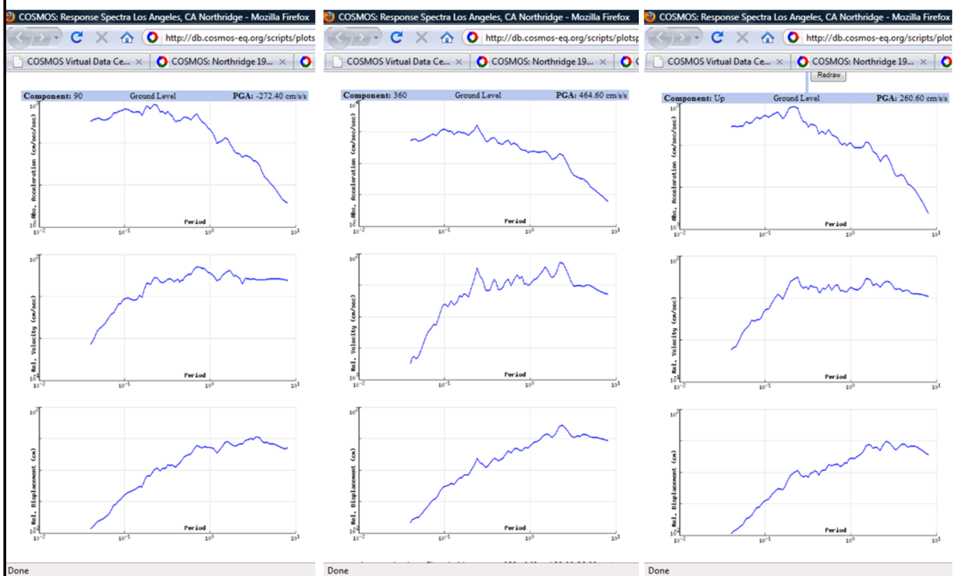

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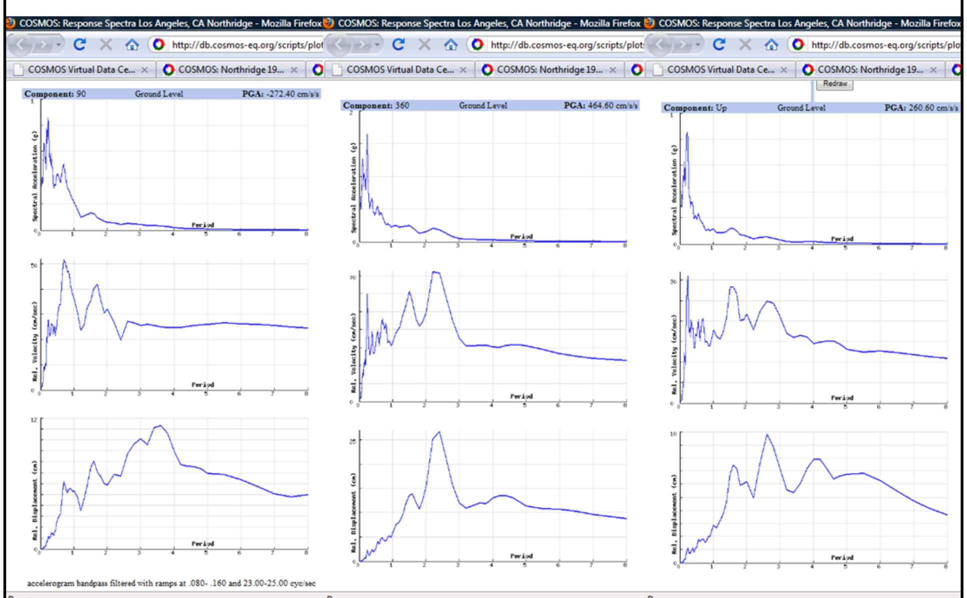

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Elastic Design Spectrum

Use recorded ground motions (available)

Use ground motions recorded at similar sites:

- Magnitude of earthquake
- Distance of site form earthquake fault
- Fault mechanism
- Local Soil Conditions
- Geology/travel path of seismic waves

Motions recorded at the same location. For design, we need an envelope. One way is to take the average (mean) of these values (use statistics to define curves for mean and standard deviation, see next page)

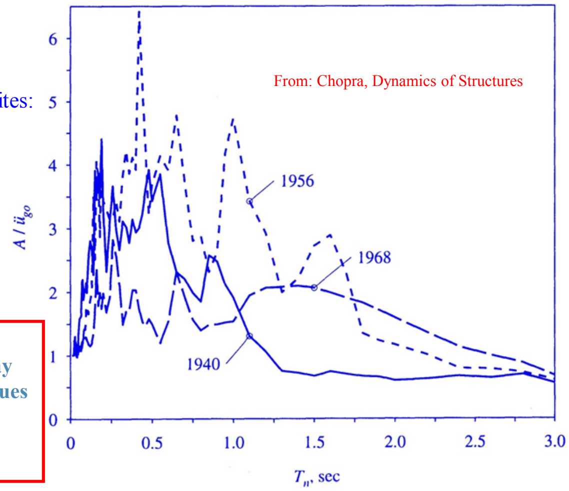


Figure 6.9.1 Response spectra for the north–south component of ground motions recorded at the Imperial Valley Irrigation District substation, El Centro, California, during earthquakes of May 18, 1940; February 9, 1956; and April 8, 1968. $\zeta = 2\%$.

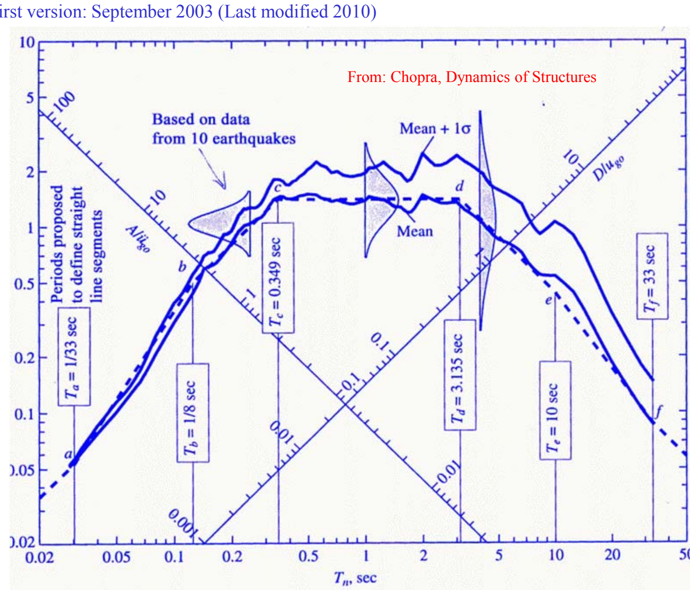
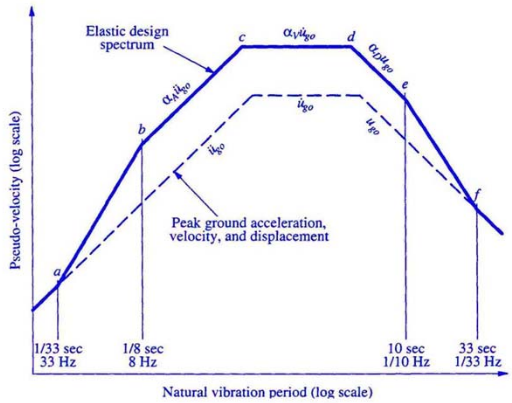


Figure 6.9.2 Mean and mean $+1\sigma$ spectra with probability distributions for V at T_n = 0.25, 1, and 4 sec; $\zeta = 5\%$. Dashed lines show an idealized design spectrum. (Based on numerical data from R. Riddell and N. M. Newmark, 1979.)

The response spectra used to derive this design spectrum would typically share common characteristics of relevance to the geographic location of interest such as: distance from causative fault, soil profile at the site, expected earthquake magnitude, fault rupture mechanism, and so forth, ..

Mean response spectrum is smooth relative to any of the original contributing spectra

As an alternative Empirical approach, the periods shown on the this Figure and the values in the Table can be used to construct a Median elastic design spectrum (or a Median + one Sigma), where Sigma is the Standard Deviation. **For any geographic location, these spectra are built based on an estimate of peak ground acceleration, velocity, and displacement as this location.**



From: Chopra, Dynamics of Structures

Figure 6.9.3 Construction of elastic design spectrum.

The periods and values in Table 6.9.1

were selected to give a good match

to a statistical curve such as Figure 6.9.2

based on an ensemble

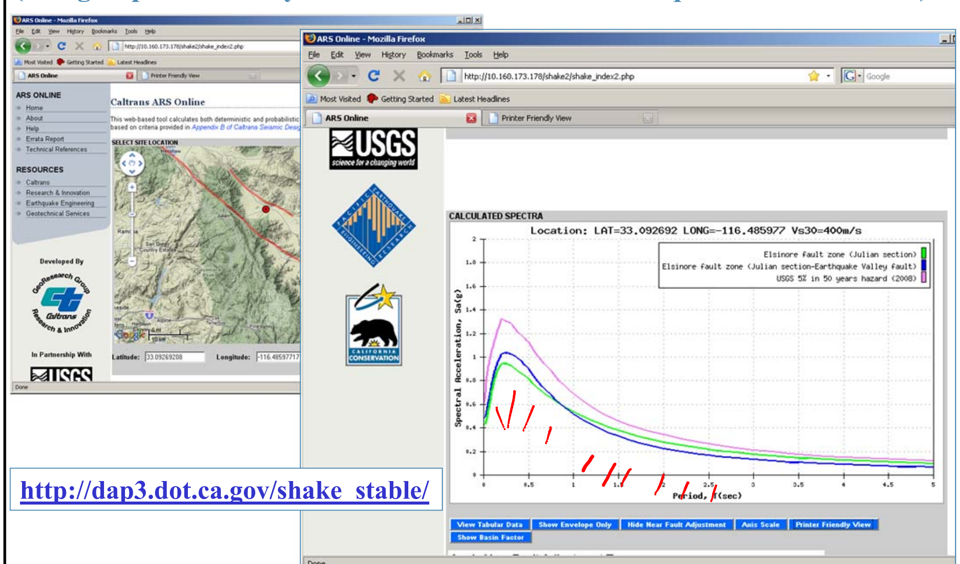
of 50 earthquakes on competent soils.

TABLE 6.9.1 AMPLIFICATION FACTORS: ELASTIC DESIGN SPECTRA

Damping, ζ (%)	Median (50 percentile)			One Sigma (84.1 percentile)		
	α_A	α_V	α_D	α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.59	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

Source: N. M. Newmark and W. J. Hall, *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, Calif., 1982, pp. 35 and 36.

(Design Spectrum may include more than one Earthquake fault scenario)



http://dap3.dot.ca.gov/shake_stable/

About Caltrans ARS Online

This web-based tool calculates both deterministic and probabilistic acceleration response spectra for any location in California based on criteria provided in *Appendix B of Caltrans Seismic Design Criteria*