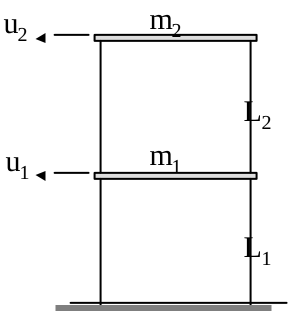
SE 180

Final Project

November 5, 2002
SE 180: Earthquake Engineering

2 Story Shear Frame



Given:

- m_1
- L_1
- L_2
- EI
- ω_1
- ω_2

Solve for m_2

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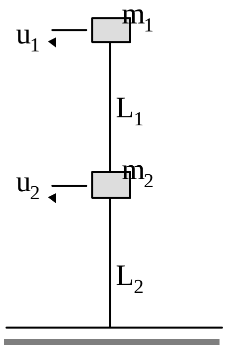
2 Story Bending Beam



Given:

- m_1
- L_1
- L_2
- EI
- ω_1
- ω_2

Solve for m_2

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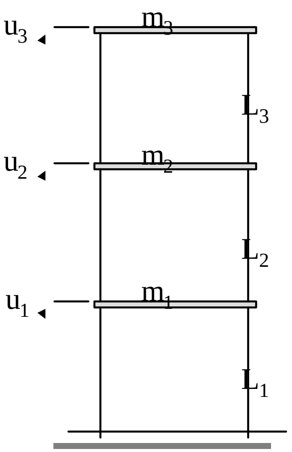
3 Story Shear Frame

Given:

- m_1
- m_2
- L_1
- L_2
- L_3
- EI
- ω_1
- ω_2
- ω_3

Solve for m_3

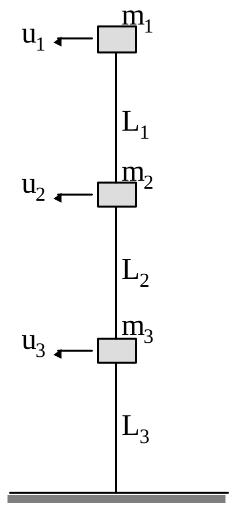


3 Story Bending Beam

Given:

- m_1
- m_2
- $L_1=L_2=L_3=L$
- EI
- ω_1
- ω_2
- ω_3

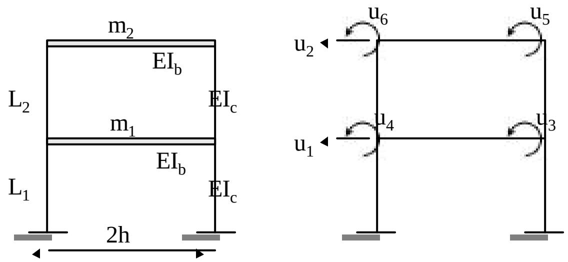
Solve for m_3



Part 1: Determining the unknown mass of your structure

Step 1: Assemble the mass and stiffness matrices for your structure

For Example: Find m_2, given m_1, $L_1 = L_2 = h$, EI_c, & EI_b



Step 1: Mass and Stiffness Matrices (continued)

Example (continued):

$$m = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k = \begin{bmatrix} \frac{48 \cdot E \cdot I_c}{h^3} & \frac{-24 \cdot E \cdot I_c}{h^3} & 0 & 0 & \frac{6 \cdot E \cdot I_c}{h^2} & \frac{6 \cdot E \cdot I_c}{h^2} \\ \frac{-24 \cdot E \cdot I_c}{h^3} & \frac{24 \cdot E \cdot I_c}{h^3} & \frac{-6 \cdot E \cdot I_c}{h^2} & \frac{-6 \cdot E \cdot I_c}{h^2} & \frac{-6 \cdot E \cdot I_c}{h^2} & \frac{-6 \cdot E \cdot I_c}{h^2} \\ 0 & \frac{-6 \cdot E \cdot I_c}{h^2} & \frac{8 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} & \frac{2 \cdot E \cdot I_c}{h} & 0 \\ 0 & \frac{-6 \cdot E \cdot I_c}{h^2} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} & \frac{8 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} & 0 & \frac{2 \cdot E \cdot I_c}{h} \\ \frac{6 \cdot E \cdot I_c}{h^2} & \frac{-6 \cdot E \cdot I_c}{h^2} & \frac{2 \cdot E \cdot I_c}{h} & 0 & \frac{4 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} \\ \frac{6 \cdot E \cdot I_c}{h^2} & \frac{-6 \cdot E \cdot I_c}{h^2} & 0 & \frac{2 \cdot E \cdot I_c}{h} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} & \frac{4 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} \end{bmatrix}$$

Determining the unknown mass of your structure

Step 2: Perform Static Condensation (if necessary)

$$[k] = \begin{bmatrix} k_{tt} & k_{t0} \\ k_{0t} & k_{00} \end{bmatrix} \qquad \hat{k}_{tt} = k_{tt} - k_{0t}^T k_{00}^{-1} k_{0t}$$

Example (continued):

$$k_{tt} = \begin{bmatrix} \frac{48 \cdot E \cdot I_c}{h^3} & \frac{-24 \cdot E \cdot I_c}{h^3} \\ \frac{-24 \cdot E \cdot I_c}{h^3} & \frac{24 \cdot E \cdot I_c}{h^3} \end{bmatrix}$$

$$k_{0t} = \begin{bmatrix} 0 & -\frac{6 \cdot E \cdot I_c}{h^2} \\ 0 & -\frac{6 \cdot E \cdot I_c}{h^2} \\ \frac{6 \cdot E \cdot I_c}{h^2} & -\frac{6 \cdot E \cdot I_c}{h^2} \\ \frac{6 \cdot E \cdot I_c}{h^2} & -\frac{6 \cdot E \cdot I_c}{h^2} \end{bmatrix}$$

$$k_{00} = \begin{bmatrix} \frac{8 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} & \frac{2 \cdot E \cdot I_c}{h} & 0 \\ \frac{2 \cdot E \cdot I_b}{2 \cdot h} & \frac{8 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} & 0 & \frac{2 \cdot E \cdot I_c}{h} \\ \frac{2 \cdot E \cdot I_c}{h} & 0 & \frac{4 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} \\ 0 & \frac{2 \cdot E \cdot I_c}{h} & \frac{2 \cdot E \cdot I_b}{2 \cdot h} & \frac{4 \cdot E \cdot I_c}{h} + \frac{4 \cdot E \cdot I_b}{2 \cdot h} \end{bmatrix}$$

Step 2: Static Condensation (continued)

Example (continued):

$$\hat{k}_{tt} = k_{tt} - k_{\alpha}^{T} k_{00}^{-1} k_{\alpha} = \begin{bmatrix} 24 \cdot E \cdot Ic \cdot \frac{(32 \cdot Ic^2 + 63 \cdot Ic \cdot Ib + 18 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} & -24 \cdot E \cdot Ic \cdot \frac{(10 \cdot Ic^2 + 27 \cdot Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} \\ -24 \cdot E \cdot Ic \cdot \frac{(10 \cdot Ic^2 + 27 \cdot Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} & 24 \cdot E \cdot Ic \cdot \frac{(4 \cdot Ic^2 + 18 \cdot Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} \end{bmatrix}$$

Where $\hat{k}_{tt}$ is the condensed stiffness matrix

Determining the unknown mass of your structure

Step 3: Solve the Eigen-value problem to determine the determinant of

$$\left[[k] - ?^2 [m] \right]$$

Example (continued):

$$[k] - ?^2 [m] = \begin{bmatrix} 24 \cdot E \cdot Ic \cdot \frac{(32 Ic^2 + 63 \cdot Ic \cdot Ib + 18 \cdot Ib^2)}{[(28 Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} - \omega^2 \cdot m1 & -24 \cdot E \cdot Ic \cdot \frac{(10 \cdot Ic^2 + 27 \cdot Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} \\ -24 \cdot E \cdot Ic \cdot \frac{(10 \cdot Ic^2 + 27 \cdot Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} & 24 \cdot E \cdot Ic \cdot \frac{(4 \cdot Ic^2 + 18 Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} - \omega^2 \cdot m2 \end{bmatrix}$$

Step 3: Eigen-value Problem (continued)

Example (continued):

$$\left| \left[[k] - ?^2 [m] \right] \right| =$$

= [illegible]

Determining the unknown mass of your structure

Step 4: Plug the given values for m_1, L_1, L_2, EI, & ω_1 into $\left| [k] - ?^2 [m] \right| = 0$, and solve for the unknown mass.

Example (continued):

Given:

m_1 = 0.2 kg
L_1 = 0.3048 m
L_2 = 0.3048 m
EI = 1.0889242 n-m^2
ω_1 = 12.566 rad/s
ω_2 = 86.013rad/s

Step 4: Solve for unknown m (continued)

Example (continued):

Plug m_1, L_1, L_2, EI, & ω_1 into $\left|[k] - ?^2[m]\right| = 0$

Solve for m_2

$$m_2 = 24 E \cdot Ic \cdot \frac{\left(24 \cdot E \cdot Ic^3 + 216 Ic^2 \cdot E \cdot Ib - 4 \cdot Ic^2 \cdot \omega_1^2 \cdot m1 \cdot h^3 - 18 Ic \cdot \omega_1^2 \cdot m1 \cdot h^3 \cdot Ib + 216 Ic \cdot E \cdot Ib^2 - 9 \cdot \omega_1^2 \cdot m1 \cdot h^3 \cdot Ib^2\right)}{\left[h^3 \cdot \omega_1^2 \cdot \left(768 E \cdot Ic^3 - 28 Ic^2 \cdot \omega_1^2 \cdot m1 \cdot h^3 + 1512 Ic^2 \cdot E \cdot Ib + 432 Ic \cdot E \cdot Ib^2 - 36 Ic \cdot \omega_1^2 \cdot m1 \cdot h^3 \cdot Ib - 9 \cdot \omega_1^2 \cdot m1 \cdot h^3 \cdot Ib^2\right)\right]}$$

$m_2 = 0.949$ kg

Determining the unknown mass of your structure

Step 5: Perform a self-check on the value of the mass you just found

Plug the newly determined mass along with the remaining ω_i into $\left|[k] - ?^2[m]\right|$ and verify that the determinant is still equal to zero

Example (continued):

$$m_2 = 24 E \cdot Ic \cdot \frac{\left(24 \cdot E \cdot Ic^3 + 216 Ic^2 \cdot E \cdot Ib - 4 \cdot Ic^2 \cdot \omega_2^2 \cdot m1 \cdot h^3 - 18 Ic \cdot \omega_2^2 \cdot m1 \cdot h^3 \cdot Ib + 216 Ic \cdot E \cdot Ib^2 - 9 \cdot \omega_2^2 \cdot m1 \cdot h^3 \cdot Ib^2\right)}{\left[h^3 \cdot \omega_2^2 \cdot \left(768 E \cdot Ic^3 - 28 \cdot Ic^2 \cdot \omega_2^2 \cdot m1 \cdot h^3 + 1512 Ic^2 \cdot E \cdot Ib + 432 Ic \cdot E \cdot Ib^2 - 36 Ic \cdot \omega_2^2 \cdot m1 \cdot h^3 \cdot Ib - 9 \cdot \omega_2^2 \cdot m1 \cdot h^3 \cdot Ib^2\right)\right]}$$

$m_2 = 0.949$ kg (same as with ω_1 – OK!)

Part 2: Modal Analysis

Step 1: Determine the Mode Shapes f_n

$$\mathbf{(k - ?^2 m)u = 0} \qquad \text{Equation 1}$$

Continuing with the Eigen-value problem solution (again, Matlab does this, or by hand for a 2-dof system), for each $?_n$ we get an associated f_n ← mode shape. To do this (for each identified $?_n$), go ahead and substitute this $?_n$ for $?$ in Eq. 1 above. Upon this substitution, you can solve for the corresponding vector **u**, the components of which defines the mode shape f_n .

Step 1: Mode Shapes (continued)

For the Previous example:

$\omega_1 = 12.566$ rad/s

$$\begin{bmatrix} 24 E \cdot Ic \cdot \frac{\left(32 Ic^2 + 63 \cdot Ic \cdot Ib + 18 Ib^2\right)}{\left[\left(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2\right) \cdot h^3\right]} - \omega 1^2 \cdot m1 & -24 \cdot E \cdot Ic \cdot \frac{\left(10 Ic^2 + 27 \cdot Ic \cdot Ib + 9 \cdot Ib^2\right)}{\left[\left(28 \cdot Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2\right) \cdot h^3\right]} \\ -24 \cdot E \cdot Ic \cdot \frac{\left(10 \cdot Ic^2 + 27 \cdot Ic \cdot Ib + 9 \cdot Ib^2\right)}{\left[\left(28 Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2\right) \cdot h^3\right]} & 24 \cdot E \cdot Ic \cdot \frac{\left(4 \cdot Ic^2 + 18 Ic \cdot Ib + 9 \cdot Ib^2\right)}{\left[\left(28 Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2\right) \cdot h^3\right]} - \omega 1^2 \cdot m2 \end{bmatrix} = \begin{bmatrix} 1.397 \cdot 10^3 & -581.566 \\ -581.566 & 242.065 \end{bmatrix}$$

$$\begin{bmatrix} 1.397 \cdot 10^3 & -581.566 \\ -581.566 & 242.065 \end{bmatrix} \cdot \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let $\phi_{11} = 1.0$; therefore, $\phi_{21} = 2.402$

Step 1: Mode Shapes (continued)

Example (continued):

$\omega_2 = 86.011$ rad/s

$$\begin{bmatrix} 24 E \cdot Ic \cdot \frac{(32 \cdot Ic^2 + 63 Ic \cdot Ib + 18 \cdot Ib^2)}{[(28 Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} - \omega 2^2 \cdot m1 & -24 E \cdot Ic \cdot \frac{(10 Ic^2 + 27 Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} \\ -24 \cdot E \cdot Ic \cdot \frac{(10 Ic^2 + 27 Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} & 24 \cdot E \cdot Ic \cdot \frac{(4 \cdot Ic^2 + 18 Ic \cdot Ib + 9 \cdot Ib^2)}{[(28 \cdot Ic^2 + 36 \cdot Ic \cdot Ib + 9 \cdot Ib^2) \cdot h^3]} - \omega 2^2 \cdot m2 \end{bmatrix} = \begin{bmatrix} -50.933 & -581.566 \\ -581.566 & -6.629 \cdot 10^3 \end{bmatrix}$$

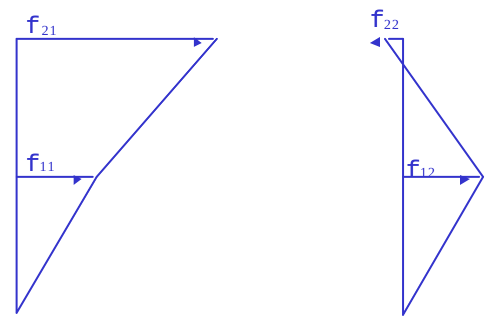
$$\begin{bmatrix} -50.933 & -581.566 \\ -581.566 & -6.629 \cdot 10^3 \end{bmatrix} \cdot \begin{bmatrix} \phi_{12} \\ \phi_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let $\phi_{12} = 1.0$; therefore, $\phi_{22} = -0.0876$

Step 1: Mode Shapes (continued)

Example (continued):

First Mode Second Mode



Part 2: Modal Analysis

Step 2: Determine the Modal Participation Factors $\frac{L_i}{M_i}$

$$M_i = \sum_{j=1}^{NDOF} m_j f_{ji}^2$$

$$L_i = \sum_{j=1}^{NDOF} m_j f_{ji}$$

$$f_i = \begin{bmatrix} f_{1i} \\ f_{2i} \\ . \\ . \\ . \\ f_{NDOF\ i} \end{bmatrix}$$

Step 2: Modal Participation Factors (continued)

Example (continued):

$$M_i = \sum_{j=1}^{NDOF} m_j f_{ji}^2$$

$M_1 = m_1 f_{11}^2 + m_2 f_{21}^2 = (0.2)(1)^2 + (0.949)(2.402)^2 = 5.67535$ kg

$M_2 = m_1 f_{12}^2 + m_2 f_{22}^2 = (0.2)(1)^2 + (0.949)(-0.0876)^2 = 0.20728$ kg

$$L_i = \sum_{j=1}^{NDOF} m_j f_{ji}$$

$L_1 = m_1 f_{11} + m_2 f_{21} = (0.2)(1) + (0.949)(2.402) = 2.4795$ kg

$L_2 = m_1 f_{12} + m_2 f_{22} = (0.2)(1) + (0.949)(-0.0876) = 0.11687$ kg

Step 2: Modal Participation Factors (continued)

Example (continued):

$$\frac{L_1}{M_1} = \frac{2.4795\ \text{kg}}{5.67535\ \text{kg}} = 0.437$$

$$\frac{L_2}{M_2} = \frac{0.11687\ \text{kg}}{0.20728\ \text{kg}} = 0.563$$

Part 2: Modal Analysis

Step 3: Determine K_i

$$K_i = ?_i^2 M_i$$

$$K_1 = ?_1^2 M_1 = (12.566)^2 (5.67535) = 896.1625$$

$$K_2 = ?_2^2 M_2 = (86.011)^2 (0.20728) = 1533.435$$

Part 2: Modal Analysis

Step 4: Add Damping

Now, you can add any modal damping you wish (which is another big plus, since you control the damping in each mode individually). If you choose ζ_i = 0.02 or 0.05, the equations become:

$$\ddot{q}_i + 2x_i ?_i \dot{q}_i + ?_i^2 q_i = -\frac{L_i}{M_i}\ddot{u}_g, \quad i = 1, 2, \ldots \text{NDOF}$$

Part 2: Modal Analysis

Step 5: Solve for $q_i(t)$

Solve for $q_i(t)$ in the above uncoupled equations (using a SDOF-type program), and the final solution is obtained from:

$$\mathbf{u} = \mathbf{F}\,\mathbf{q}$$
$$\dot{\mathbf{u}} = \mathbf{F}\,\dot{\mathbf{q}}$$
$$\ddot{\mathbf{u}} = \mathbf{F}\,\ddot{\mathbf{q}}$$
$$\ddot{\mathbf{u}}^t = \ddot{\mathbf{u}} + \mathbf{1}\ddot{u}_g$$

Step 5: Solve for $q_i(t)$ (continued)

We will solve for $q_i(t)$ using a modified version of the spreadsheet for solving for the response of a SDOF system using Newmark's Method

Part 3: Spreadsheet for Modal Analysis

Step-By-Step Procedure For Setting Up A Spreadsheet For Using Newmark's Method and Modal Analysis To Solve For The Response Of A Multi-Degree Of Freedom (MDOF) System

Start with the equation of motion for a linear multi-degree of freedom system with base ground excitation:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{1}\ddot{u}_g$$

Using Modal Analysis, we can rewrite the original coupled matrix equation of motion as a set of un-coupled equations.

$$\ddot{q}_i + 2??\,\dot{q}_i + ?_i^2 q_i = -\frac{L_i}{M_i}\ddot{u}_g \quad , \; i = 1, 2, \ldots, \text{NDOF}$$

with initial conditions of $d_i(t=0) = d_{io}$ and $v_i(t=0) = v_{io}$

Note that total acceleration or absolute acceleration will be

$$\ddot{q}_{i\,abs} = \ddot{q}_i + \ddot{u}_g$$

We can solve each one separately (as a SDOF system), and compute histories of q_i and their time derivatives. To compute the system response, plug the q vector back into $\mathbf{u} = \mathbf{F}\,\mathbf{q}$ and get the u vector (and the same for the time derivatives to get velocity and acceleration).

The beauty here is that there is no matrix operations involved, since the matrix equation of motion has become a set of un-coupled equation, each including only one generalized coordinate q_n.

In the spreadsheet, we will solve each mode in a separate worksheet.

Step 1 - Define System Properties and Initial Conditions for First Mode

(A) Begin by setting up the cells for the Mass, Stiffness, and Damping of the SDOF System (Fig. 1). These values are known.

(B) Set up the cells for the modal participation factor $\frac{L_i}{M_i}$ and mode shape f_i (Fig. 1). These values must be determined in advance using Modal Analysis.

(C) Calculate the Natural Frequency of the SDOF system using the equation

$$?_i = \sqrt{K_i/M_i} \qquad \text{(Equation 1)}$$

Step 1 (continued)

Note: If the system damping is given in terms of the Modal Damping Ratio (ζ_i) then the Damping (C_i) can be calculated using the equation:

$$C_i = 2\,\zeta_i\,\omega_i\,M_i \qquad \text{(Equation 2)}$$

(D) Set up the cells for the 2 Newmark Coefficients α & β (Fig. 1), which will allow for performing

a) the Average Acceleration Method, use $a = \frac{1}{2}$ and $ß = \frac{1}{6}$.

b) the Linear Acceleration Method, use $a = \frac{1}{2}$ and $ß = \frac{1}{4}$.

(E) Set up cells (Fig. 1) for the initial displacement and velocity (d_o and v_o respectively)

Step 1 (continued)

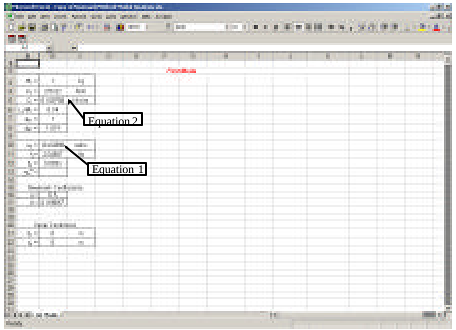


Figure 1: Spreadsheet After Completing Step 1

Step 2 – Set Up Columns for Solving The Equation of Motion Using Newmark's Method

Place a cell (Fig. 2) for the time increment (Δt).

Place columns (Fig. 2) for the time, base excitation, applied force divided by mass, relative acceleration, relative velocity, and relative displacement.

Step 2 (continued)

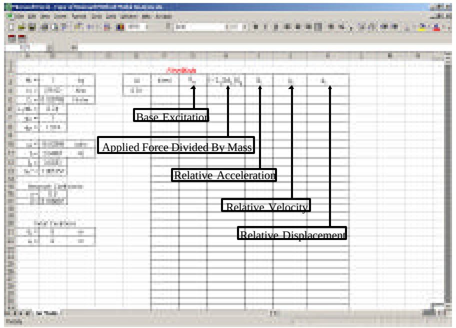


Figure 2: Spreadsheet After Completing Step 2

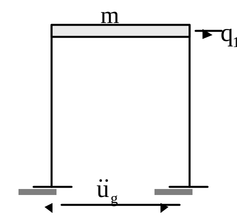
Step 3 – Enter the Time t & Applied Force f(t) into the Spreadsheet

$$t_{i+1} = t_i + ?\,t \qquad \text{(Equation 3) (Fig. 3)}$$

For the earthquake problem (acceleration applied to base of the structure), the applied force divided by the mass is calculated using:

$$\frac{f_i(t)}{M_1} = -\frac{L_1}{M_1}\ddot{u}_{g_i} \qquad \text{(Equation 4) (Fig. 3)}$$

where, $\ddot{u}_{g_i}$ is the applied base acceleration at step i. (Typically this is the base excitation time history)



Check the units of the input motion file. They must be compatible with the units of the mass, stiffness, and damping!

Step 3 (continued)

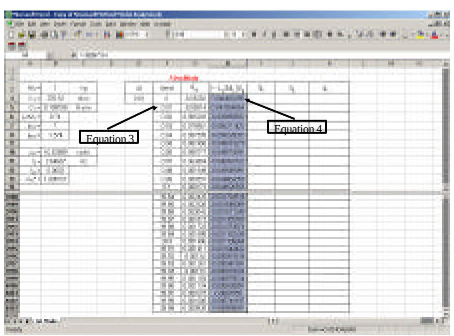


Figure 3: Spreadsheet After Completing Step 3

Step 4 – Compute Initial Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration

(A) The Initial Relative Displacement and Relative Velocity are known from the initial conditions (Fig. 4).

$$q(t=0) = d_o \qquad \text{(Equation 5)}$$

$$\dot{q}(t=0) = v_o \qquad \text{(Equation 6)}$$

(B) The Initial Relative Acceleration (Fig. 4) is calculated using

$$\ddot{q}(t=0) = -\frac{Li}{Mi}\ddot{u}_g - 2??\,v_o - ?^2 d_o \qquad \text{(Equation 7)}$$

Step 4 (continued)

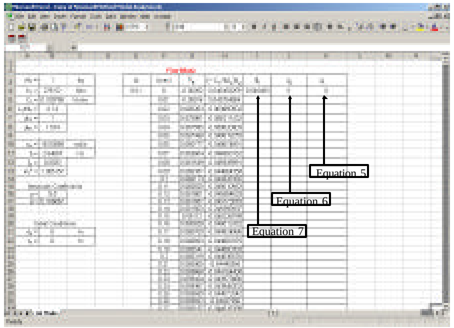


Figure 4: Spreadsheet After Completing Step 4

Step 5 – Compute Incremental Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration At Each Time Step (Fig. 5)

(A)

$$\ddot{q}_{i+1} = \frac{\left[-\frac{L_1}{M_1}\ddot{u}_{g\,i+1} - C_1\left(\frac{?t}{2}\ddot{q}_i + \dot{q}_i\right) - K_1\left(\frac{1}{2}?t^2(1-2ß)\ddot{q}_i + ?t\,\dot{q}_i + q_i\right)\right]}{m_1^*} \quad \text{(Equation 8)}$$

$$\dot{q}_{i+1} = \ddot{q}_i ?t(1-a) + \ddot{q}_{i+1} ?t\,a + \dot{q}_i \quad \text{(Equation 9)}$$

$$q_{i+1} = \ddot{q}_i \frac{?t^2}{2}(1-2ß) + \ddot{q}_{i+1} ?t^2 ß + \dot{q}_i ?t + q_i \quad \text{(Equation 10)}$$

Where, the effective mass, $m_1^* = M_1 + C_1 ?t\,a + K_1 ?t^2 ß$

Step 5 (continued)

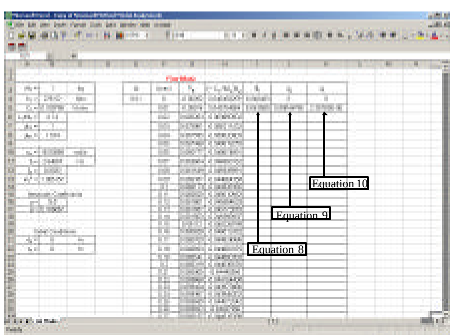


Figure 5: Spreadsheet with values for the Relative Acceleration, Relative Velocity, and Relative Displacement at Time Step 1

Step 5 (continued)

(B) Then, highlight columns I, J, & K and rows 4 through to the last time step (in this example 4003) and "Fill Down" (Ctrl+D).

See Figures 6 and 7.

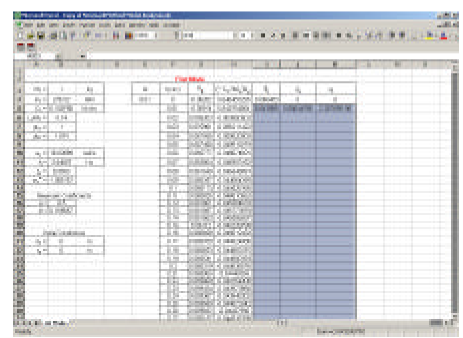


Figure 6: Highlighted Cells

Step 5 (continued)

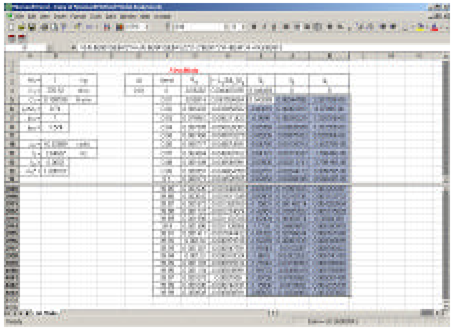


Figure 7: Spreadsheet After "Filling Down" Columns I through K

Step 6 – Create Additional Worksheet for Second Mode

Make a copy of the "1st Mode" worksheet by right clicking on the "1st Mode" tab and selecting "Move or Copy" (Fig. 8)

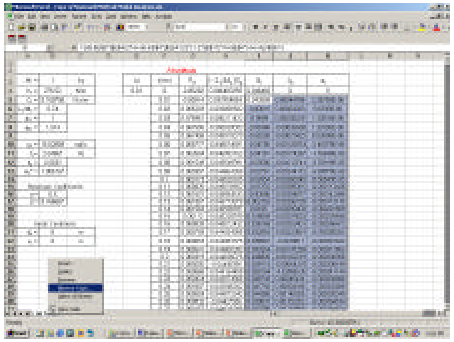


Figure 8: Creating a Copy of 1st Mode Worksheet

Step 6 (continued)

Then check the box for "Create a copy" and click on "OK" button (Fig. 9)

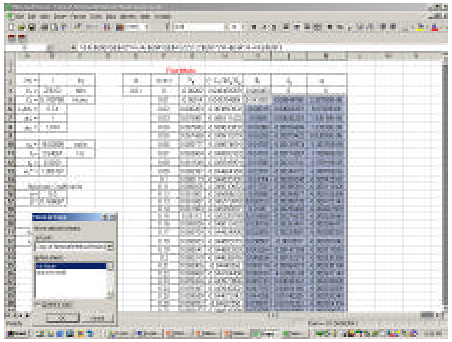


Figure 9: Creating a Copy of 1st Mode Worksheet

Step 6 (continued)

Rename this worksheet by right clicking on the "1st Mode (2)" tab and selecting "Rename". Rename this worksheet "2nd Mode" (Fig. 10)

Enter the appropriate values for M_2, K_2, C_2, $\frac{L_2}{M_2}$, f_2, d_o, and v_o (Fig. 10).

Step 6 (continued)

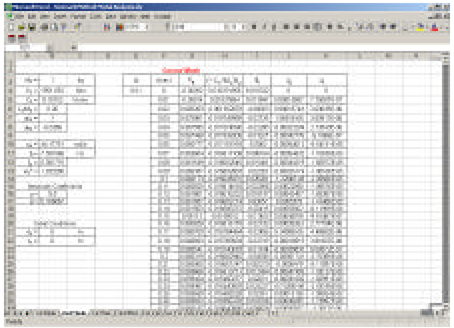


Figure 10: Worksheet for Second Mode

Step 7 – Repeat Step 6 for Additional Modes

Step 8 – Determine the Response at Each of the Floors

Determine the Response of the first floor using the equations:

$$\mathbf{u} = \mathbf{F}\,\mathbf{q}$$

$$\dot{\mathbf{u}} = \mathbf{F}\,\dot{\mathbf{q}}$$

$$\ddot{\mathbf{u}} = \mathbf{F}\,\ddot{\mathbf{q}}$$

Step 8 (continued)

For example for a 2DOF structure, the first floor response is (Fig. 11)

$$u_1 = f_{11} q_1 + f_{12} q_2 \qquad \text{(Equation 11)}$$

$$\dot{u}_1 = f_{11} \dot{q}_1 + f_{12} \dot{q}_2 \qquad \text{(Equation 12)}$$

$$\ddot{u}_1 = f_{11} \ddot{q}_1 + f_{12} \ddot{q}_2 \qquad \text{(Equation 13)}$$

Step 8 (continued)

and the second floor response is (Fig. 12)

$$u_2 = f_{21} q_1 + f_{22} q_2 \qquad \text{(Equation 14)}$$

$$\dot{u}_2 = f_{21} \dot{q}_1 + f_{22} \dot{q}_2 \qquad \text{(Equation 15)}$$

$$\ddot{u}_2 = f_{21} \ddot{q}_1 + f_{22} \ddot{q}_2 \qquad \text{(Equation 16)}$$

The first floor absolute acceleration is

$$\ddot{u}_1^T = \ddot{u}_1 + \ddot{u}_g \qquad \text{(Equation 17)}$$

The second floor absolute acceleration is

$$\ddot{u}_2^T = \ddot{u}_2 + \ddot{u}_g \qquad \text{(Equation 18)}$$

Step 8 (continued)

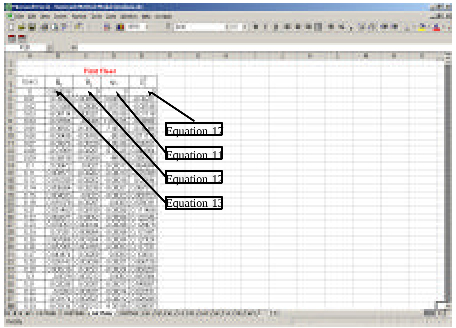


Figure 11: First Floor Response

Step 8 (continued)

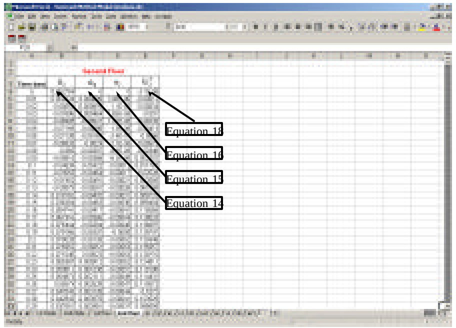


Figure 12: Second Floor Response