SE 180 Earthquake Engineering

October 2, 2002

STEP-BY-STEP PROCEDURE FOR SETTING UP A SPREADSHEET FOR USING Newmark's Method To Solve For The Response Of A Single Degree Of Freedom (SDOF) System

We are solving $ma + cv + kd = -m\ddot{u}_g$ with initial conditions of $d(t = 0) = d_o$ and $v(t = 0) = v_o$ Note that total acceleration or absolute acceleration will be $u_{abs} = a + \ddot{u}_g$

Step 1 - Define System Properties and Initial Conditions

(A) Begin by setting up the cells for the Mass, Stiffness, and Damping of the SDOF System (Fig. 1). These values are known.

(B) Calculate the Natural Frequency of the SDOF system using the equation

$$\omega = \sqrt{k/m}$$
 (Equation 1)

Note: If the system damping is given in terms of the Damping Ratio (ζ) then the Damping (c) can be calculated using the equation:

$$c=2 \zeta \omega m$$
 (Equation 2)

(C) Set up the cells for the 2 Newmark Coefficients $\alpha \& \beta$ (Fig. 1), which will allow for performing

a) the Average Acceleration Method, use
$$\alpha = \frac{1}{2}$$
 and $\beta = \frac{1}{4}$.

b) the Linear Acceleration Method, use $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{6}$.

Ahmed Elgamal Michael Fraser (D) Set up cells (Fig. 1) for the initial displacement and velocity (d_o and v_o respectively)



Figure 1: Spreadsheet After Completing Step 1

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Step 2 – Set Up Columns for Solving The Equation of Motion Using Newmark's Method

Figure 2: Spreadsheet After Completing Step 2

Place a cell (Fig. 2) for the time increment (Δt).

Place columns (Fig. 2) for the time, base excitation, applied force, relative acceleration, relative velocity, relative displacement, and absolute acceleration.

Step 3 – Enter the Time t & Applied Force f(t) into the Spreadsheet

 $t_{i+1} = t_i + \Delta t$ (Equation 3) (Fig. 3)

For the earthquake problem (acceleration applied to base of the structure), the applied force is calculated using:

$$f_i(t) = -m\ddot{u}_{g_i}$$
 (Equation 4) (Fig. 3)

where, \ddot{u}_{g_i} is the applied base acceleration at step i. (Typically this is the base excitation time history)

Check the units of the input motion file. They must be compatible with the units of the mass, stiffness, and damping!



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10	Newr	nark Coeff	icients			0.07	0.065777	-0.065776609							
11	a . =	0.5				0.08	0.063504	-0.06350426							
12	β =	0.166667				0.09	0.061549	-0.061548637							
13						0.1	0.060357	-0.060357241							
14						0.11	0.060173	-0.060173196							
15	Ini	tial Conditi	ions			0.12	0.060825	-0.060825071							
16	d _o =	0	m			0.13	0.061601	-0.061600855							
17	v _0 =	0	m			0.14	0.061857	-0.061856592							
18						0.15	0.061563	-0.061562969							
19						0.16	0.06112	-0.061119998							
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Figure 3: Spreadsheet After Completing Step 3

Ahmed Elgamal Michael Fraser Step 4 – Compute Initial Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration

(A) The Initial Relative Displacement and Relative Velocity are known from the initial conditions (Fig. 4).

 $d(t = 0) = d_o$ (Equation 5)

 $v(t = 0) = v_0$ (Equation 6)

(B) The Initial Relative Acceleration (Fig. 4) is calculated using

$$a(t=0) = \frac{f(t)}{m} - 2\zeta \omega v_o - \omega_1^2 d_o \qquad (Equation 7)$$

(C) The Initial Absolute Acceleration (Fig. 4) is computed using

$$\ddot{u}_{abs} = a_o - \frac{f(t)}{m}$$
 (Equation 8)

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Figure 4: Spreadsheet After Completing Step 4

Step 5 – Compute Incremental Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration At Each Time Step (Fig. 5)

$$a_{i+1} = \frac{\left[-m\ddot{u}_{g_{i+1}} - c\left(\frac{\Delta t}{2}a_i + v_i\right) - k\left(\frac{1}{2}\Delta t^2(1 - 2\beta)a_i + \Delta tv_i + d_i\right)\right]}{m^*}$$
 (Equation 9)

$$\Delta t^2$$

$$\mathbf{d}_{i+1} = \mathbf{a}_i \frac{\Delta t^2}{2} (1 - 2\beta) + \mathbf{a}_{i+1} \Delta t^2 \beta + \mathbf{v}_i \Delta t + \mathbf{d}_i$$
 (Equation 11)

$$\ddot{u}_{abs_{i+1}} = a_{i+1} + u_{g_{i+1}}$$

 $\mathbf{v}_{i+1} = \mathbf{a}_i \Delta t (1 - \alpha) + \mathbf{a}_{i+1} \Delta t \alpha + \mathbf{v}_i$

(Equation 12)

(Equation 10)

Where, the effective mass, $m^* = m + c\Delta t \alpha + k\Delta t^2 \beta$

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8	ζ=	0.02				0.05	0.067595	-0.067594598							
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12	β =	0.166667				0.09	0.061549	-0.061548637							
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15	in		ions			0.12	0.060825	-0.060825071							
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17	v _o =	U	m			0.14	0.061857	-0.061856592		E C	10				
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Figure 5: Spreadsheet with values for the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration at Time Step 1

(B) Then, highlight columns H, I, J, & K and rows 4 through to the last time step (in this example 1562) and "Fill Down" (Ctrl+D). See Figures 6 and 7.

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Figure 6: Highlighted Cells

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3	k =	411.887	N/m		0.01	0	0	0	0	0	0	0				
4	c =	0.8118	N-s/m			0.01	-0.06282	0.062815215	0.062136	0.00031068	1.0356E-06	-0.00068		-		
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6	@ =	20.295	rad/s			0.03	0.005203	-0.005203381	-0.01342	0.00110901	1.7751E-05	-0.00821				
7	f=	3.2300495				0.04	0.0759614	-0.075961381	-0.08755	0.0006042	2.6935E-05	-0.01158				
8	ζ=	0.02				0.05	0.067595	-0.067594598	-0.07924	-0.00022975	2.8738E-05	-0.01165				
9						0.06	0.067458	-0.067458485	-0.07592	-0.00100559	2.2533E-05	-0.00846				
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14						0.11	0.060173	-0.060173196	-0.0155	-0.00344395	-0.00010166	0.044669				_
15	Init	ial Conditi	ons			0.12	0.060825	-0.060825071	-0.00168	-0.00352986	-0.00013665	0.059148				
16	d _o =	0	m			0.13	0.061601	-0.061600855	0.011985	-0.00347832	-0.0001718	0.073586				
17	v o =	0	m			0.14	0.061857	-0.061856592	0.025564	-0.00329057	-0.00020576	0.08742				
18						0.15	0.061563	-0.061562969	0.038535	-0.00297008	-0.00023717	0.100098				
19						0.16	0.06112	-0.061119998	0.04998	-0.0025275	-0.00026475	0.1111				
20						0.17	0.060828	-0.060827611	0.059148	-0.00198187	-0.00028738	0.119975				
21						0.18	0.060709	-0.06070944	0.06566	-0.00135783	-0.00030413	0.126369				•
3993						39.9	0.001598	-0.001597755	0.022623	-0.00032502	-5.8164E-05	0.024221				-
3994						39.91	0.001496	-0.00149579	0.023406	-9.4876E-05	-6.027E-05	0.024902				
3995						39.92	0.001411	-0.001411394	0.023211	0.00013821	-6.0052E-05	0.024622				
3996						39.93	0.00134	-0.001340291	0.022059	0.00036456	-5.7529E-05	0.023399				
3997						39.94	0.001281	-0.001280529	0.020006	0.00057489	-5.2814E-05	0.021287				
3998						39.95	0.00123	-0.001229762	0.017146	0.00076065	-4.6113E-05	0.018376				
3999						39.96	0.001183	-0.001183263	0.013606	0.00091441	-3.7708E-05	0.014789				
4000						39.97	0.001134	-0.001133771	0.009543	0.00103015	-2.7951E-05	0.010677				
4001						39.98	0.001075	-0.001074999	0.005133	0.00110353	-1.7246E-05	0.006208				
4002						39.99	0.001006	-0.001005672	0.000559	0.00113199	-6.0306E-06	0.001565				
4003						40	0.000928	-0.000927961	-0.00399	0.00111482	5.2414E-06	-0.00306			_	
	► ► \Shee	t1 / abs acc	/ vel / disp	/	1		1		•						лb	ĿĔ
Read	у															

Figure 7: Spreadsheet After "Filling Down" Columns H through K