October 2, 2002

STEP-BY-STEP PROCEDURE FOR SETTING UP A SPREADSHEET FOR USING NEWMARK'S METHOD TO SOLVE FOR THE RESPONSE OF A SINGLE DEGREE OF FREEDOM (SDOF) SYSTEM

We are solving $ma + cv + kd = -m\ddot{u}_g$
with initial conditions of $d(t = 0) = d_o$ and $v(t = 0) = v_o$
Note that total acceleration or absolute acceleration will be $u_{abs} = a + \ddot{u}_g$

Step 1 - Define System Properties and Initial Conditions

(A) Begin by setting up the cells for the Mass, Stiffness, and Damping of the SDOF System (Fig. 1). These values are known.

(B) Calculate the Natural Frequency of the SDOF system using the equation

$$\omega = \sqrt{k/m} \qquad \text{(Equation 1)}$$

Note: If the system damping is given in terms of the Damping Ratio (ζ) then the Damping (c) can be calculated using the equation:

$$c = 2\,\zeta\,\omega\, m \qquad \text{(Equation 2)}$$

(C) Set up the cells for the 2 Newmark Coefficients α & β (Fig. 1), which will allow for performing

a) the Average Acceleration Method, use $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{4}$.

b) the Linear Acceleration Method, use $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{6}$.

(D) Set up cells (Fig. 1) for the initial displacement and velocity (d_o and v_o respectively)

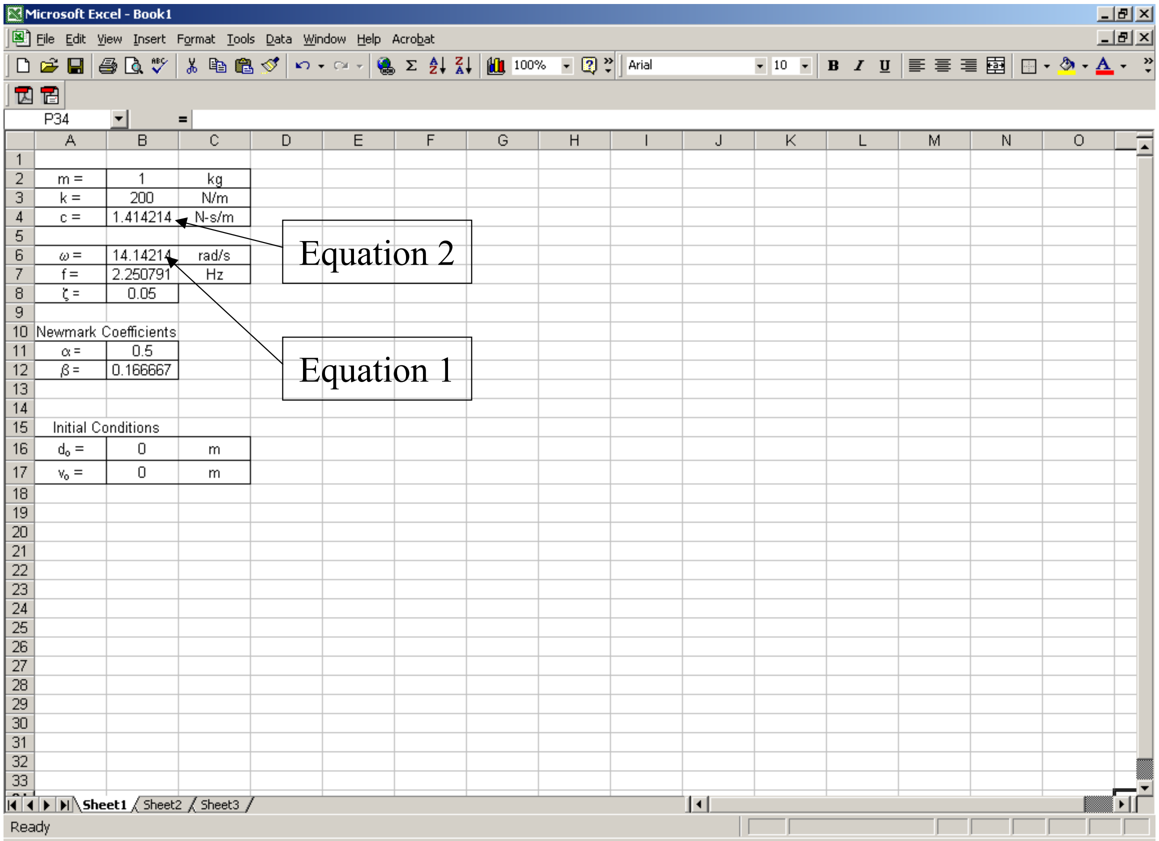


Figure 1: Spreadsheet After Completing Step 1

Step 2 – Set Up Columns for Solving The Equation of Motion Using Newmark's Method

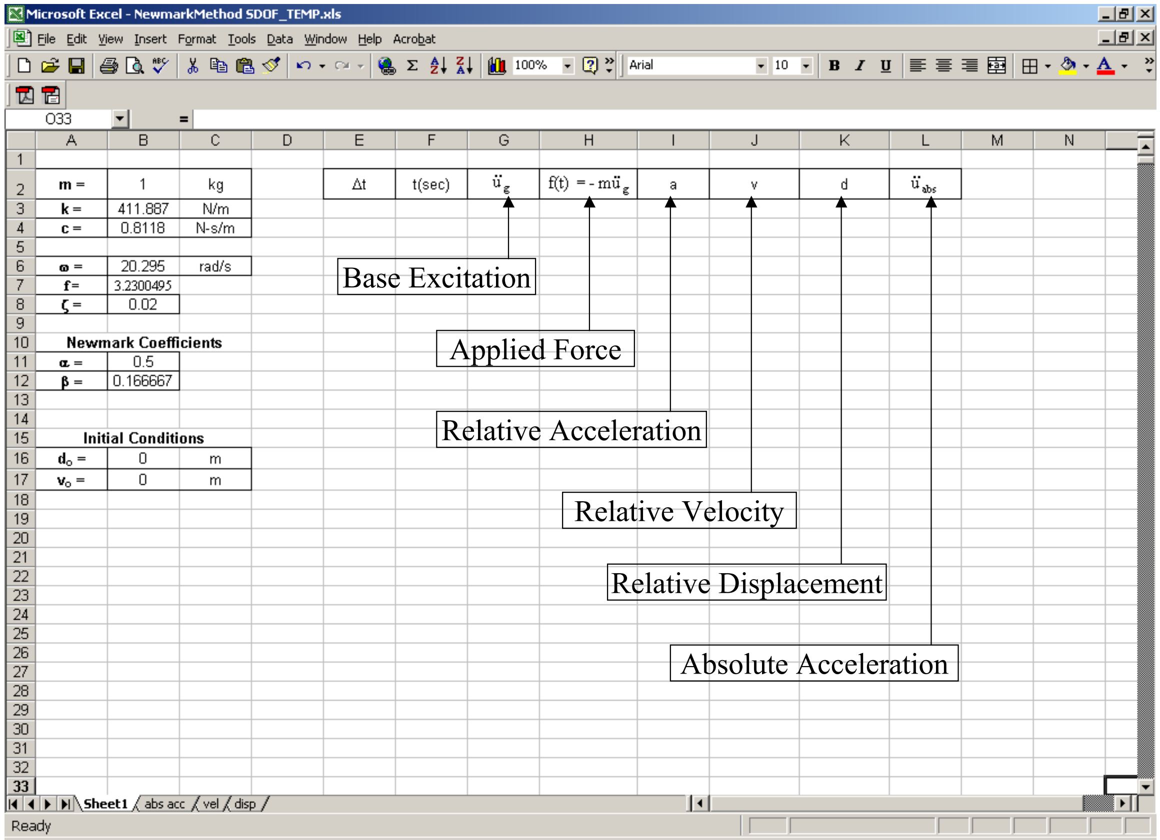


Figure 2: Spreadsheet After Completing Step 2

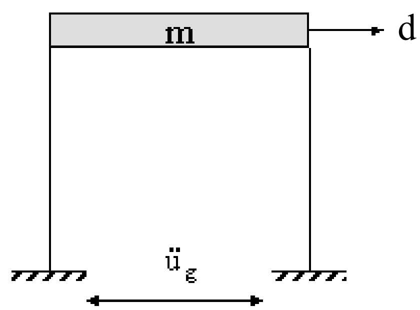
Place a cell (Fig. 2) for the time increment (Δt).
Place columns (Fig. 2) for the time, base excitation, applied force, relative acceleration, relative velocity, relative displacement, and absolute acceleration.

Step 3 – Enter the Time t & Applied Force f(t) into the Spreadsheet

$t_{i+1} = t_i + \Delta t$ (Equation 3) (Fig. 3)

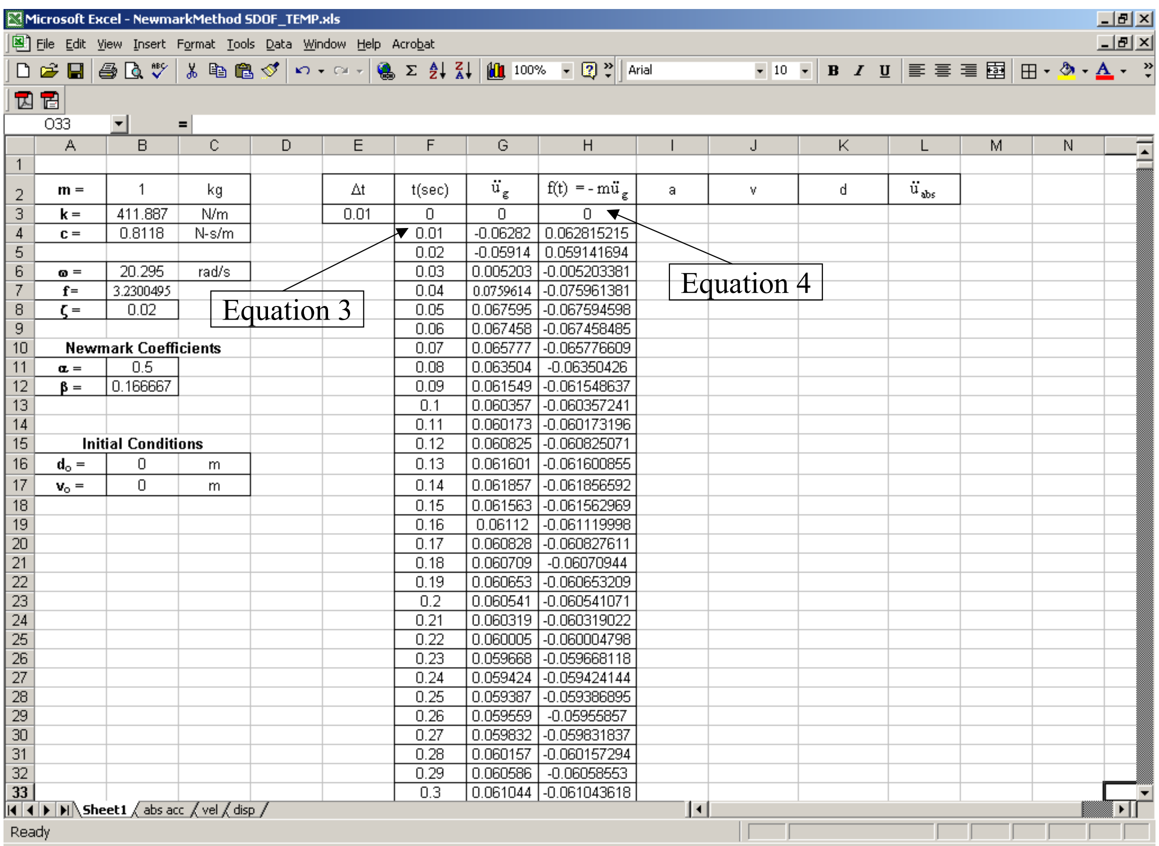
For the earthquake problem (acceleration applied to base of the structure), the applied force is calculated using:

$f_i(t) = -m\ddot{u}_{g_i}$ (Equation 4) (Fig. 3)



where, $\ddot{u}_{g_i}$ is the applied base acceleration at step i. (Typically this is the base excitation time history)

Check the units of the input motion file. They must be compatible with the units of the mass, stiffness, and damping!



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	m =	1	kg		Δt	t(sec)	$\ddot{u}_g$	$f(t) = -m\ddot{u}_g$	a	v	d	$\ddot{u}_{abs}$
3	k =	411.887	N/m		0.01	0	0	0				
4	c =	0.8118	N-s/m			0.01	-0.06282	0.062815215				
5						0.02	-0.05914	0.059141694				
6	ω =	20.295	rad/s			0.03	0.005203	-0.005203381				
7	f=	3.2300495				0.04	0.0759614	-0.075961381				
8	ζ =	0.02				0.05	0.067595	-0.067594598				
9						0.06	0.067458	-0.067458485				
10	Newmark Coefficients					0.07	0.065777	-0.065776609				
11	α =	0.5				0.08	0.063504	-0.06350426				
12	β =	0.166667				0.09	0.061549	-0.061548637				
13						0.1	0.060357	-0.060357241				
14						0.11	0.060173	-0.060173196				
15	Initial Conditions					0.12	0.060825	-0.060825071				
16	d_o =	0	m			0.13	0.061601	-0.061600855				
17	v_o =	0	m			0.14	0.061857	-0.061856592				
18						0.15	0.061563	-0.061562969				
19						0.16	0.06112	-0.061119998				
20						0.17	0.060828	-0.060827611				
21						0.18	0.060709	-0.06070944				
22						0.19	0.060653	-0.060653209				
23						0.2	0.060541	-0.060541071				
24						0.21	0.060319	-0.060319022				
25						0.22	0.060005	-0.060004798				
26						0.23	0.059668	-0.059668118				
27						0.24	0.059424	-0.059424144				
28						0.25	0.059387	-0.059386895				
29						0.26	0.059559	-0.05955857				
30						0.27	0.059832	-0.059831837				
31						0.28	0.060157	-0.060157294				
32						0.29	0.060586	-0.06058553				
33						0.3	0.061044	-0.061043618				

Figure 3: Spreadsheet After Completing Step 3

Step 4 – Compute Initial Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration

(A) The Initial Relative Displacement and Relative Velocity are known from the initial conditions (Fig. 4).

$$d(t = 0) = d_o \quad \text{(Equation 5)}$$

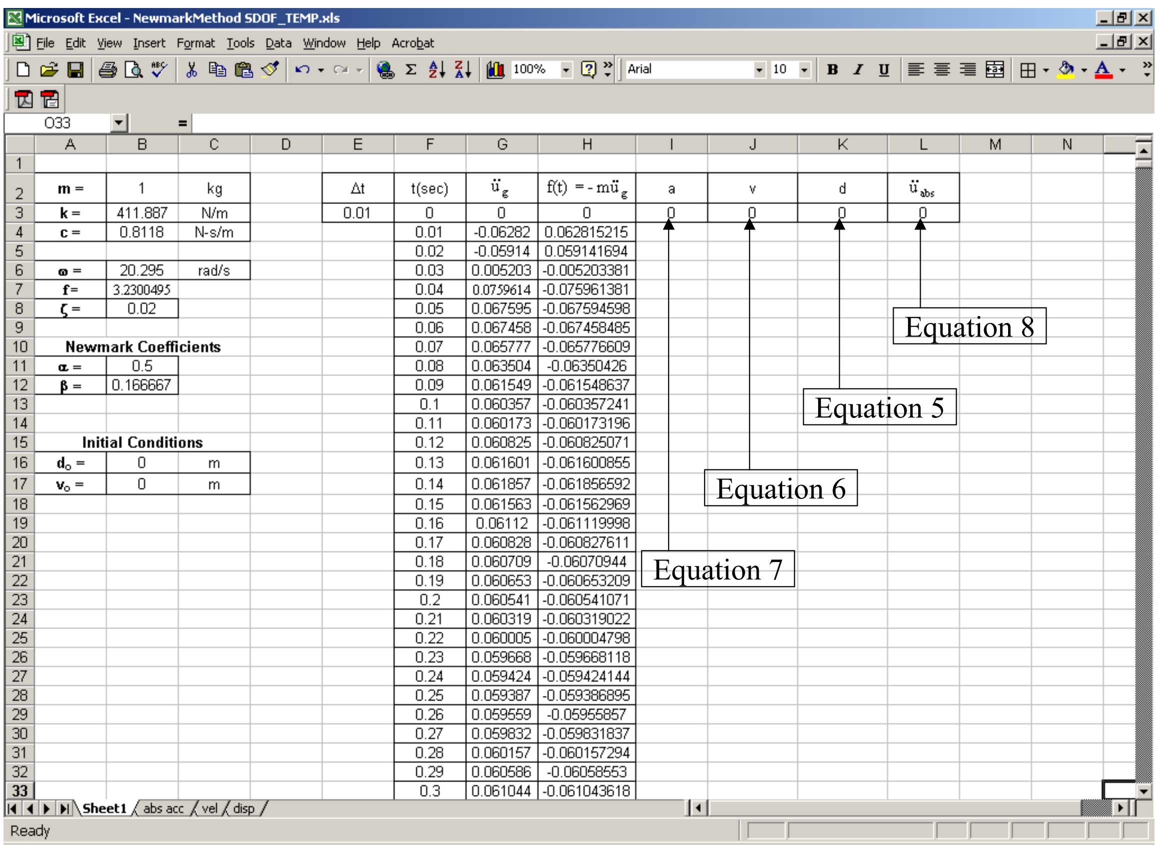
$$v(t = 0) = v_o \quad \text{(Equation 6)}$$

(B) The Initial Relative Acceleration (Fig. 4) is calculated using

$$a(t = 0) = \frac{f(t)}{m} - 2\zeta\omega v_o - \omega_1^2 d_o \qquad \text{(Equation 7)}$$

(C) The Initial Absolute Acceleration (Fig. 4) is computed using

$$\ddot{u}_{abs} = a_o - \frac{f(t)}{m} \qquad \text{(Equation 8)}$$



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	**m =**	1	kg		Δt	t(sec)	$\ddot{u}_g$	$f(t) = -m\ddot{u}_g$	a	v	d	$\ddot{u}_{abs}$
3	**k =**	411.887	N/m		0.01	0	0	0	0	0	0	0
4	**c =**	0.8118	N-s/m			0.01	-0.06282	0.062815215				
5						0.02	-0.05914	0.059141694				
6	**ω =**	20.295	rad/s			0.03	0.005203	-0.005203381				
7	**f=**	3.2300495				0.04	0.0759614	-0.075961381				
8	**ζ =**	0.02				0.05	0.067595	-0.067594598				
9						0.06	0.067458	-0.067458485				
10	**Newmark Coefficients**					0.07	0.065777	-0.065776609				
11	**α =**	0.5				0.08	0.063504	-0.06350426				
12	**β =**	0.166667				0.09	0.061549	-0.061548637				
13						0.1	0.060357	-0.060357241				
14						0.11	0.060173	-0.060173196				
15	**Initial Conditions**					0.12	0.060825	-0.060825071				
16	**d_o =**	0	m			0.13	0.061601	-0.061600855				
17	**v_o =**	0	m			0.14	0.061857	-0.061856592				
18						0.15	0.061563	-0.061562969				
19						0.16	0.06112	-0.061119998				
20						0.17	0.060828	-0.060827611				
21						0.18	0.060709	-0.06070944				
22						0.19	0.060653	-0.060653209				
23						0.2	0.060541	-0.060541071				
24						0.21	0.060319	-0.060319022				
25						0.22	0.060005	-0.060004798				
26						0.23	0.059668	-0.059668118				
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28						0.25	0.059387	-0.059386895				
29						0.26	0.059559	-0.05955857				
30						0.27	0.059832	-0.059831837				
31						0.28	0.060157	-0.060157294				
32						0.29	0.060586	-0.06058553				
33						0.3	0.061044	-0.061043618				

Figure 4: Spreadsheet After Completing Step 4

Step 5 – Compute Incremental Values of the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration At Each Time Step (Fig. 5)

(A)

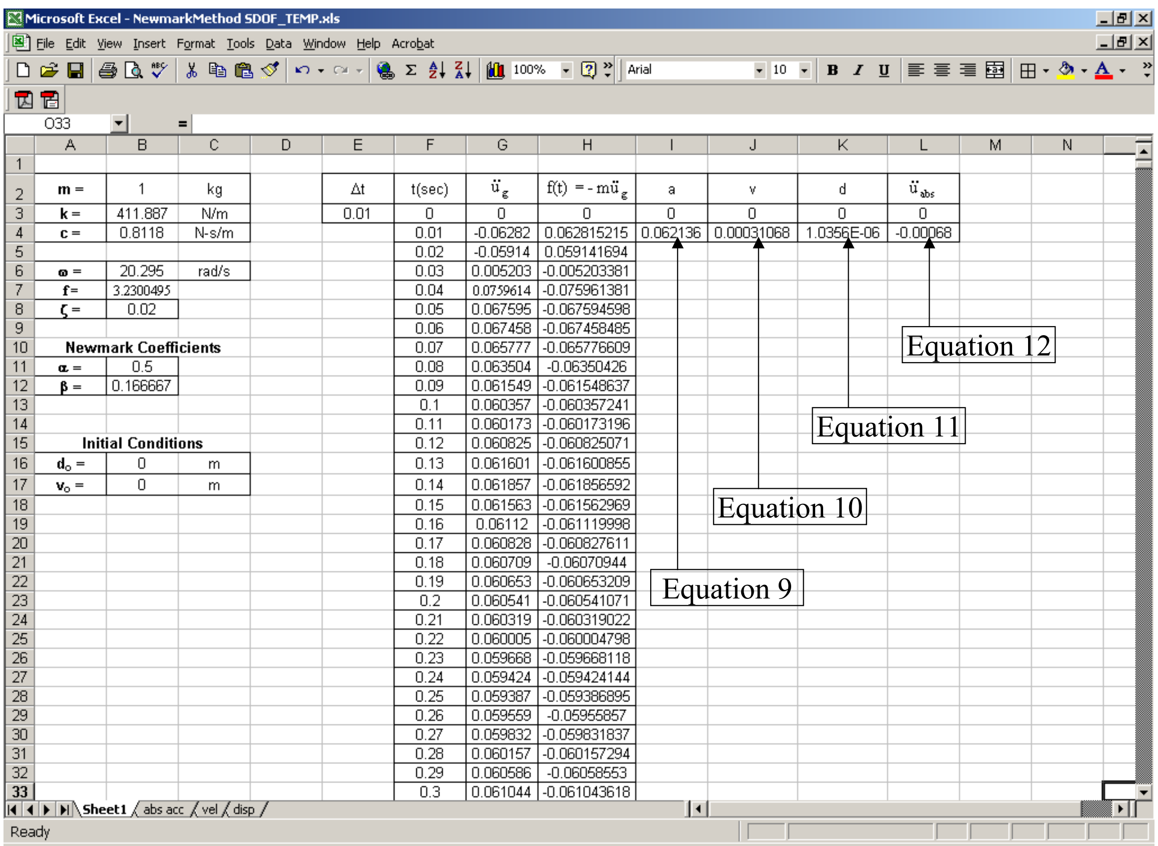
$$a_{i+1} = \frac{\left[-m\ddot{u}_{g\,i+1} - c\left(\frac{\Delta t}{2}a_i + v_i\right) - k\left(\frac{1}{2}\Delta t^2(1-2\beta)a_i + \Delta t v_i + d_i\right)\right]}{m^*} \quad \text{(Equation 9)}$$

$$v_{i+1} = a_i\Delta t(1-\alpha) + a_{i+1}\Delta t\alpha + v_i \quad \text{(Equation 10)}$$

$$d_{i+1} = a_i\frac{\Delta t^2}{2}(1-2\beta) + a_{i+1}\Delta t^2\beta + v_i\Delta t + d_i \quad \text{(Equation 11)}$$

$$\ddot{u}_{abs\,i+1} = a_{i+1} + u_{g\,i+1} \quad \text{(Equation 12)}$$

Where, the effective mass, $m^* = m + c\Delta t\alpha + k\Delta t^2\beta$



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	m =	1	kg		Δt	t(sec)	$\ddot{u}_g$	f(t) = - m$\ddot{u}_g$	a	v	d	$\ddot{u}_{abs}$
3	k =	411.887	N/m		0.01	0	0	0	0	0	0	0
4	c =	0.8118	N-s/m			0.01	-0.06282	0.062815215	0.062136	0.00031068	1.0356E-06	-0.00068
5						0.02	-0.05914	0.059141694				
6	ω =	20.295	rad/s			0.03	0.005203	-0.005203381				
7	f=	3.2300495				0.04	0.0759614	-0.075961381				
8	ζ =	0.02				0.05	0.067595	-0.067594598				
9						0.06	0.067458	-0.067458485				
10	Newmark Coefficients					0.07	0.065777	-0.065776609				
11	α =	0.5				0.08	0.063504	-0.06350426				
12	β =	0.166667				0.09	0.061549	-0.061548637				
13						0.1	0.060357	-0.060357241				
14						0.11	0.060173	-0.060173196				
15	Initial Conditions					0.12	0.060825	-0.060825071				
16	d_o =	0	m			0.13	0.061601	-0.061600855				
17	v_o =	0	m			0.14	0.061857	-0.061856592				
18						0.15	0.061563	-0.061562969				
19						0.16	0.06112	-0.061119998				
20						0.17	0.060828	-0.060827611				
21						0.18	0.060709	-0.06070944				
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23						0.2	0.060541	-0.060541071				
24						0.21	0.060319	-0.060319022				
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27						0.24	0.059424	-0.059424144				
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30						0.27	0.059832	-0.059831837				
31						0.28	0.060157	-0.060157294				
32						0.29	0.060586	-0.06058553				
33						0.3	0.061044	-0.061043618				

Figure 5: Spreadsheet with values for the Relative Acceleration, Relative Velocity, Relative Displacement, and Absolute Acceleration at Time Step 1

(B) Then, highlight columns H, I, J, & K and rows 4 through to the last time step (in this example 1562) and “Fill Down” (Ctrl+D). See Figures 6 and 7.

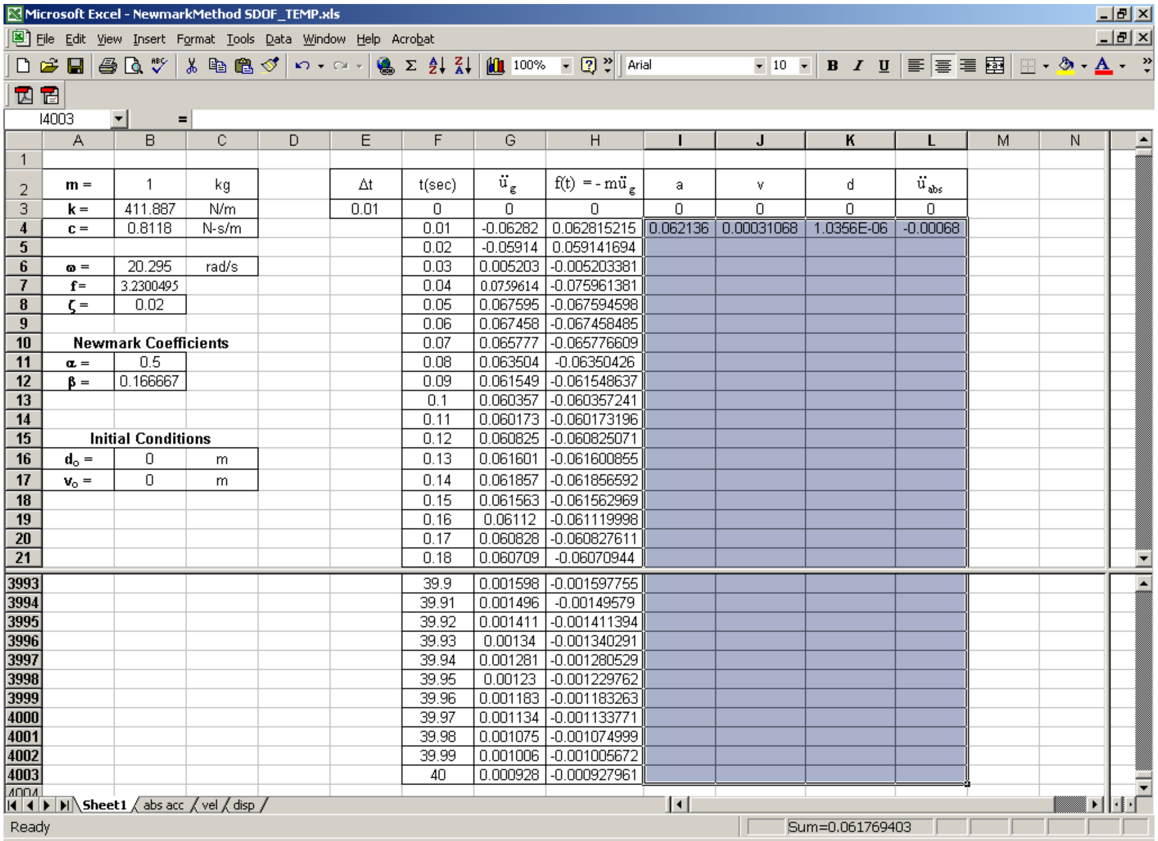
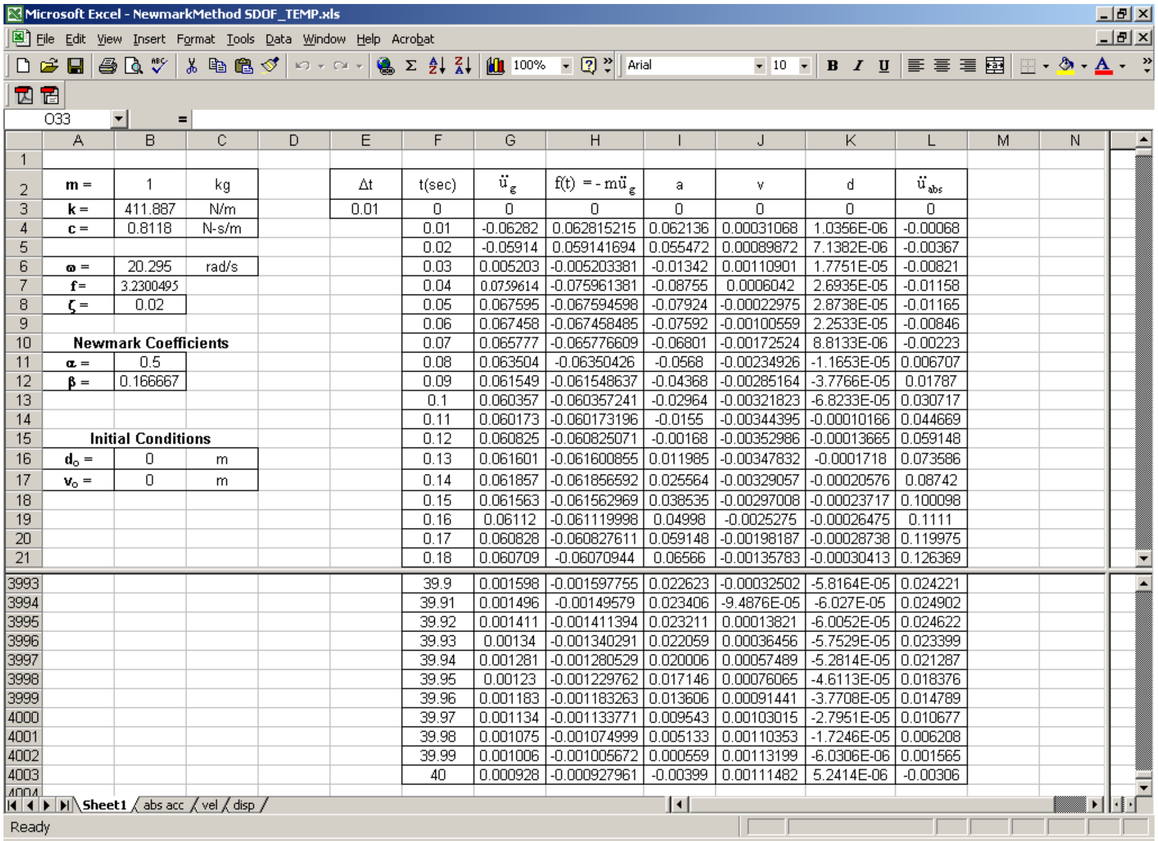


Figure 6: Highlighted Cells

Ahmed Elgamal
Michael Fraser



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	**m =**	1	kg		Δt	t(sec)	$\ddot{u}_g$	f(t) = - m$\ddot{u}_g$	a	v	d	$\ddot{u}_{abs}$
3	**k =**	411.887	N/m		0.01	0	0	0	0	0	0	0
4	**c =**	0.8118	N-s/m			0.01	-0.06282	0.062815215	0.062136	0.00031068	1.0356E-06	-0.00068
5						0.02	-0.05914	0.059141694	0.055472	0.00089872	7.1382E-06	-0.00367
6	**ω =**	20.295	rad/s			0.03	0.005203	-0.005203381	-0.01342	0.00110901	1.7751E-05	-0.00821
7	**f=**	3.2300495				0.04	0.0759614	-0.075961381	-0.08755	0.0006042	2.6935E-05	-0.01158
8	**ζ =**	0.02				0.05	0.067595	-0.067594598	-0.07924	-0.00022975	2.8738E-05	-0.01165
9						0.06	0.067458	-0.067458485	-0.07592	-0.00100559	2.2533E-05	-0.00846
10	**Newmark Coefficients**					0.07	0.065777	-0.065776609	-0.06801	-0.00172524	8.8133E-06	-0.00223
11	**α =**	0.5				0.08	0.063504	-0.06350426	-0.0568	-0.00234926	-1.1653E-05	0.006707
12	**β =**	0.166667				0.09	0.061549	-0.061548637	-0.04368	-0.00285164	-3.7766E-05	0.01787
13						0.1	0.060357	-0.060357241	-0.02964	-0.00321823	-6.8233E-05	0.030717
14						0.11	0.060173	-0.060173196	-0.0155	-0.00344395	-0.00010166	0.044669
15	**Initial Conditions**					0.12	0.060825	-0.060825071	-0.00168	-0.00352986	-0.00013665	0.059148
16	**d_o =**	0	m			0.13	0.061601	-0.061600855	0.011985	-0.00347832	-0.0001718	0.073586
17	**v_o =**	0	m			0.14	0.061857	-0.061856592	0.025564	-0.00329057	-0.00020576	0.08742
18						0.15	0.061563	-0.061562969	0.038535	-0.00297008	-0.00023717	0.100098
19						0.16	0.06112	-0.061119998	0.04998	-0.0025275	-0.00026475	0.1111
20						0.17	0.060828	-0.060827611	0.059148	-0.00198187	-0.00028738	0.119975
21						0.18	0.060709	-0.06070944	0.06566	-0.00135783	-0.00030413	0.126369
3993						39.9	0.001598	-0.001597755	0.022623	-0.00032502	-5.8164E-05	0.024221
3994						39.91	0.001496	-0.00149579	0.023406	-9.4876E-05	-6.027E-05	0.024902
3995						39.92	0.001411	-0.001411394	0.023211	0.00013821	-6.0052E-05	0.024622
3996						39.93	0.00134	-0.001340291	0.022059	0.00036456	-5.7529E-05	0.023399
3997						39.94	0.001281	-0.001280529	0.020006	0.00057489	-5.2814E-05	0.021287
3998						39.95	0.00123	-0.001229762	0.017146	0.00076065	-4.6113E-05	0.018376
3999						39.96	0.001183	-0.001183263	0.013606	0.00091441	-3.7708E-05	0.014789
4000						39.97	0.001134	-0.001133771	0.009543	0.00103015	-2.7951E-05	0.010677
4001						39.98	0.001075	-0.001074999	0.005133	0.00110353	-1.7246E-05	0.006208
4002						39.99	0.001006	-0.001005672	0.000559	0.00113199	-6.0306E-06	0.001565
4003						40	0.000928	-0.000927961	-0.00399	0.00111482	5.2414E-06	-0.00306

Figure 7: Spreadsheet After "Filling Down" Columns H through K