Class Notes from: Geotechnical Earthquake Engineering
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Ground motion parameters

Ground motion parameters are important for describing the involved characteristics of importance (i.e., amplitude, frequency content, and duration) of strong ground motions.

1 Amplitude parameters

The most common way of describing a ground motion is through the time history.

- Acceleration time history,
- Velocity time history, and
- Displacement time history

Typically, only one of these is recorded directly with the others computed form it by integration/differentiation. Note that integration produces a smoothing or filtering effect. The acceleration time history displays more high frequency content (relatively), the velocity time history displays more intermediate frequency content (relatively), and the displacement displays more low frequency content (relatively).

1.1 Peak acceleration

Peak horizontal acceleration (PHA): the largest (absolute) value of the horizontal acceleration.

Because of its relationship to inertial force, intensity-acceleration relationships can be used to estimate PHA when other information is not available.

Peak vertical acceleration (PVA): the largest (absolute) value of the vertical acceleration.

It is often assumed that the ratio of PVA to PHA is 2/3 for engineering purposes, although the ratio is quite variable. Generally, PVA/PHA is greater than 2/3 near the source and less than 2/3 at large distance.

Ground motions with high peak accelerations are usually, but not always, more destructive than motions with lower peak accelerations. Damage is also related to other characteristics (e.g., frequency content and duration).

1.2 Peak velocity

Since the velocity is more sensitive to the intermediate frequency components of the ground motion, the PHV may characterize the ground motion more accurately at intermediate frequencies than the PHA. The PHV may provide a much more accurate indication of the potential for damage in structures that are more sensitive to loading in the intermediate frequency range.

1.3 Peak displacement

Peak displacements are generally associated with the lower-frequency components of a ground motion. They are, however, often difficult to determine accurately due to signal processing errors and long period noise. They are less commonly used than peak acceleration and peak velocity.

1.4 Sustained maximum acceleration and velocity

It is the 3^{rd} or 5^{th} largest peak in an acceleration or velocity time history. Damage, in some cases, may require repeated cycles of high amplitude to develop.

2 Frequency content parameters

2.1 Ground motion spectra

Fourier spectra

Fourier transform brings a motion in the time domain to the frequency domain.

Fourier amplitude spectrum is a plot of Fourier amplitude versus frequency, showing the distribution of the amplitude of a motion with respect to frequency. It expresses the frequency content of a motion very clearly. Large earthquakes produce greater low frequency motions than smaller earthquakes.

Fourier phase spectrum is a plot of Fourier phase angle versus frequency. It describes the relative variation between the constituent harmonic signals in the motion time history.

Response spectra

The response spectrum describes the maximum response of a SDOF system to a particular input motion as a function of the natural frequency and damping ratio of the SDOF system. It provides information on the potential effects of an input motion on different structures.

- A linear response spectrum corresponds to a linear structural force displacement relationship.
- A nonlinear response spectrum corresponds to a nonlinear structural force displacement relationship.
- Acceleration response spectra: maximum acceleration response versus structural natural frequency and damping ratio.
- Velocity response spectra: maximum Velocity response versus structural natural frequency and damping ratio.

- Displacement response spectra: maximum Displacement response versus structural natural frequency and damping ratio.

2.2 Spectral parameters

Predominant period: the period corresponds to the peak Fourier amplitude.

Bandwidth: the range of frequency over which some level of Fourier amplitude is exceeded.

3 Duration

Degradation of stiffness and strength of certain types of structures and the buildup of pore water pressures in loose, saturated sand, are sensitive to the number of cycles of a ground motion.

The duration of strong ground motion is related to the time required to release the accumulated strain energy by rupture along the fault. The strong motion duration increases with earthquake magnitude.

The most commonly used definition is the bracketed duration. It is defined as the time between the first and last exceedances of a threshold acceleration.

Estimation of Ground motion parameters

1. Development of predicative relationships

Predicative relationships usually express ground motion parameters as functions of earthquake magnitude, distance, source characteristics, site characteristics, etc. A typical predicative relationship may have the form

$$\ln Y = C_1 + C_2 M + C_3 M^{C_4} + C_5 \ln[R + C_6 \exp(C_7 M)] + C_8 R + f(source) + f(site)$$
$$\sigma_{\ln Y} = C_9$$

where Y is the ground motion parameter of interest, M the magnitude of the earthquake, R a measure of the distance from the source to the site being considered. C_1 - C_9 are constants to be determined. The $\sigma_{\ln Y}$ term describes the uncertainty in the value of the ground motion parameter given by the predicative relationship.

2. Attenuation relationships for the estimation of peak acceleration and peak velocity

2.1 Peak acceleration

Based on world-wide data, Campbell (1981) proposed:

$$\ln PHA(g) = -4.141 + 0.868M - 1.09 \ln[R + 0.606 \exp(0.7M)]$$
$$\sigma_{\ln Y} = 0.37$$

where M is the local magnitude or surface wave magnitude (≥ 6), R is the closest distance to the fault rupture in km (≤ 50).

Campbell and Bozorgnia (1994) proposed:

$$\begin{aligned}\ln PHA(gals) = {} & 3.512 + 0.904 M_w - 1.328 \ln\sqrt{R^2 + [0.149 \exp(0.647 M_w)]^2} \\ & + (1.125 - 0.112 \ln R - 0.0957 M_w) F + (0.940 - 0.171 \ln R) S_{SR} \\ & + (0.405 - 0.222 \ln R) S_{HR}\end{aligned}$$

$$\sigma_{\ln PHA} = \begin{cases} 0.889 - 0.0691 M_w & M_w \leq 7.4 \\ 0.38 & M_w > 7.4 \end{cases}$$

where M_w is the moment magnitude, R is the closest distance to seismic rupture in km (≤ 60, with minimum values of 7.3, 5.8, 3.5, and 3.0 km for magnitudes of 5.0, 5.5, 6.0, and 6.5, respectively), F is the source term (0 for strike-slip and normal faulting, and 1 for reverse faulting), S_{SR}=1 for soft-rock sites, S_{HR}=1 for hard rock sites, and $S_{SR}=S_{HR}$=0 for alluvium sites.

Based on western North America data, Boore et al. (1993) proposed:

$$\log PHA(g) = b_1 + b_2(M_w - 6) + b_3(M_w - 6)^2 + b_4 R + b_5 \log R + b_6 G_B + b_7 G_C$$

$$G_B = \begin{cases} 0 & for \quad site \quad class \quad A \\ 1 & for \quad site \quad class \quad B \\ 0 & for \quad site \quad class \quad C \end{cases}$$

$$G_C = \begin{cases} 0 & for \quad site \quad class \quad A \\ 0 & for \quad site \quad class \quad B \\ 1 & for \quad site \quad class \quad C \end{cases}$$

where $R = \sqrt{d^2 + h^2}$, d is the closest distance to the surface projection of the fault in km and h is focal depth in km, $b_1 - b_7$ are coefficients.. Site classes are defined on the basis of the average shear wave velocity in the upper 30 m layer of the ground.

Toro et al. (1994) proposed an attenuation relationship for the mid-continental portion of the eastern North America where the continental crust is stronger and more intact than that in western North America. Young's et al. (1988) proposed an attenuation relationship for subduction zone where earthquakes generally occur at greater hypocentral depths.

2.2 Peak velocity

Joyner and Boore (1988) developed:

$$\log PHV(cm/s) = j_1 + j_2(M_w - 6) + j_3(M_w - 6)^2 + j_4 \log R + j_5 R + j_6$$

where $R = \sqrt{r_0^2 + j_7^2}$, r_0 is the shortest distance in km from the site to the vertical projection of the earthquake fault rupture on the surface of the earth, $j_1 - j_7$ are coefficients.

Effective Design Acceleration

Since, pulses of high acceleration at high frequencies induce little response in most structures, the notion of effective design acceleration, with different definitions, has been proposed.

- Benjamin and Associates (1988): peak acceleration that remains after filtering out accelerations above 8-9 Hz.
- Kennedy (1980): 25% higher than the third highest peak acceleration obtained from a filtered time history.

Power Spectrum or Power Spectral Density Function

It is used to describe the frequency content of a ground motion. It can also be used to estimate the statistical properties of a ground motion and to compute stochastic response using random vibration techniques.

Total Intensity I_0 of A Ground Motion of Duration T_d

- In time domain, I_0 is given by the area under the squared acceleration time history ($(a(t))^2$):

$$I_0 = \int_0^{T_d} [a(t)^2]dt \tag{1}$$

- In frequency domain, I_0 is given by the area under the squared Fourier amplitude spectrum ($c(\omega)^2$) divided by π:

$$I_0 = \frac{1}{\pi}\int_0^{\omega_N} [c(\omega)^2]d\omega \tag{2}$$

Where $c(\omega)$ is Fourier amplitude spectrum, and $\omega_N = \frac{2\pi}{2\Delta t}$ is the Nyquist frequency (the highest frequency in Fourier spectrum.

Parseval's theorem can show that (1) and (2) are equal. The average intensity or mean squared acceleration λ_0 is given by dividing (1) or (2) by the duration T_d.

$$\lambda_0 = \frac{1}{T_d}\int_0^{T_d}[a(t)^2]dt = \frac{1}{\pi T_d}\int_0^{\omega_N}[c(\omega)^2]d\omega \tag{3}$$

Power Spectral Density G(Ω)

G(ω) is defined such that

$$\lambda_0 = \int_0^{\omega_N} G(\omega)d\omega \tag{4}$$

From (2) and (4), G(ω) is obtained.

$$G(\omega) = \frac{c(\omega)^2}{\pi T_d} \tag{5}$$

Normalized power spectral density $G^n(\omega)$ is given by

$$G^n(\omega) = \frac{G(\omega)}{\lambda_0} = \frac{c(\omega)^2}{\pi T_d}\frac{1}{\lambda_0} \tag{6}$$

Central Frequency Ω

Power spectral density G(ω) can be used to estimate statistical properties of a ground motion, such as Central Frequency Ω. The central Frequency is a measure of the frequency where the power spectral density is concentrated.

The nth momentum of G(ω) is defined by:

$$\lambda_n = \int_0^{\omega_N} G(\omega)\omega^n d\omega \tag{7}$$

Central Frequency Ω is given by (Vanmarcke 1976)

$$\Omega = \sqrt{\frac{\lambda_2}{\lambda_0}} \tag{8}$$

The median peak acceleration $\ddot{u}_{\max}$ can also be obtained.

$$\ddot{u}_{\max} = \sqrt{2\lambda_0 \ln\left(2.8\frac{\Omega T_d}{2\pi}\right)} \tag{9}$$

Shape Factor δ

The shape factor δ is a measure of the dispersion of the power spectral density about central frequency and is given by (Vanmarcke 1976)

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \tag{10}$$

The shape factor always lies between o and 1, with higher values corresponding to larger bandwidth.

Kanai-Tajimi Parameters G_0, ξ_g, and ω_g

Individual power spectral density functions may have highly irregular shapes. Averaging a number of normalized power spectral density functions $G^n(\omega)$ for similar strong ground motions reveals a smooth characteristic shape. Kanai (1957) and Tajimi (1960) proposed a model for power spectral density based on a limited number of strong motion records.

$$G(\omega) = G_0 \frac{1 + [2\xi_g(\omega/\omega_g)]^2}{[1-(\omega/\omega_g)^2]^2 + [2\xi_g(\omega/\omega_g)]^2} \tag{11}$$

It can be seen that components with frequencies greater than $\sqrt{2}\omega_g$ is attenuated and other components is amplified.

V_{max}/a_{max}

Because of the filtering effects when integrating acceleration to obtain velocity, peak velocity and peak acceleration are usually associated with motions of different frequency. The ratio V_{max}/a_{max} is related to the frequency content of the motion. For a simple harmonic motion of period T, $V_{max}/a_{max} = T/2\pi$. For earthquake motions, V_{max}/a_{max} can be interpreted as the period of an equivalent harmonic motion, thus providing an indication of which periods of the ground motion are most significant.

rms Acceleration

$$a_{rms} = \sqrt{\frac{1}{T_d}\int_0^{T_d}[a(t)^2]dt} = \sqrt{\lambda_0} \tag{12}$$

Note that the integral is over the duration of strong motion 0-T_d. T_d depends on the method used to define strong motion duration.

Arias Intensity

$$I_a = \frac{\pi}{2g}\int_0^{\infty}[a(t)^2]dt = \sqrt{\lambda_0} \tag{13}$$

Note that the integral is over the entire duration rather than over the duration of strong motion. Arias intensity is in dependent of the method used to define strong motion duration.

Cumulative Absolute Velocity

It is the area under the absolute accelerogram, used to correlate structural damage potential.

$$CAV = \int_0^{T_d}|\,a(t)\,|\,dt \tag{14}$$

Response Spectrum Intensity

It is the area under the pseudo-velocity response spectrum between periods of 0.1 and 2.5 sec. It provides an indication of the potential response of a structure.

$$SI(\xi) = \int_{0.1}^{2.5} PSV(\xi, T)dT \tag{15}$$

Velocity Spectrum Intensity for Earth and Rockfill Dams

It is the response spectrum intensity at 5% damping.

Von Thun et al. (1988) $$VSI = \int_{0.1}^{0.5} PSV(\xi = 0.05, T)dT \tag{16}$$

Makdisi and Seed (1978)
$$VSI = \int_{0.6}^{2.0} PSV(\xi = 0.05, T)dT \tag{17}$$

Accelerogram Spectrum Intensity (Von Thun Et Al. 1988) for Concrete Dams

It is the area under the acceleration response spectrum between periods of 0.1 and 0.5 sec.

$$ASI = \int_{0.1}^{0.5} S_a(\xi = 0.05, T)dT \tag{18}$$

Effective Peak Acceleration (EPA) and Effective Peak Velocity (EPV) By ATC (1978)

They are used to minimize the influence of local spikes in response spectra.

$$EPA(\xi) = \int_{0.1}^{0.5} S_a(\xi, T)dT \tag{19}$$

$$EPV(\xi) = \frac{S_v(\xi, T = 1\sec)}{2.5} \tag{20}$$

Estimation of frequency content parameters

The frequency content of ground motions change with distance. As seismic wave travels away from the source, high frequency components are absorbed and low frequency components gradually dominate ground motions.

Estimation of Predominant Period

The predominant period increases with distance from the source. Seed et al. (1966) proposed a chart to estimate the predominant period at rock outcrops at different earthquake magnitude and site distance.

Estimation of Fourier Amplitude Spectra

$$|A(f)| = \left[CM_0 \frac{f^2}{1-(f/f_c)^2} \frac{1}{\sqrt{1+(f/f_{max})^8}} \right] \frac{e^{-\pi f R / Q(f)^{v_s}}}{R} \tag{21}$$

where f_c and f_{max} are the corner and cutoff frequencies of the Fourier amplitude spectrum in log scale. Q(f) is the frequency dependent quality factor inversely proportional to the damping ratio of the rock.

$$C = \frac{R_{\theta\phi} F V}{4\pi\rho v_s^3} \tag{22}$$

where $R_{\theta\phi}$ (≈ 0.55) accounts for the radiation pattern, F(=2) accounts for the free-surface effect, V($=\sqrt{2}/2$) accounts for partitioning the energy into two horizontal components, ρ is the density of the rock along the rupture surface, and v_s is the shear wave velocity of the rock.

If f_{max} is assumed constant for a given geographic region (15 Hz and 40 Hz are typical values for western and eastern North America, respectively), the spectra for different earthquakes are functions of the seismic moment, M_o, and f_c, which can be related by (Brune 1970)

$$f_c = 4.9\times10^6 v_s\left(\frac{\Delta\sigma}{M_o}\right)^{1/3} \tag{23}$$

Where v_s is in km/sec, M_o is in dyne·cm, and $\Delta\sigma$ is referred the stress parameter or stress drop in bars (50 and 100 bars are commonly used for sources in western and eastern North America, respectively).

Estimation of V_{max}/a_{max}

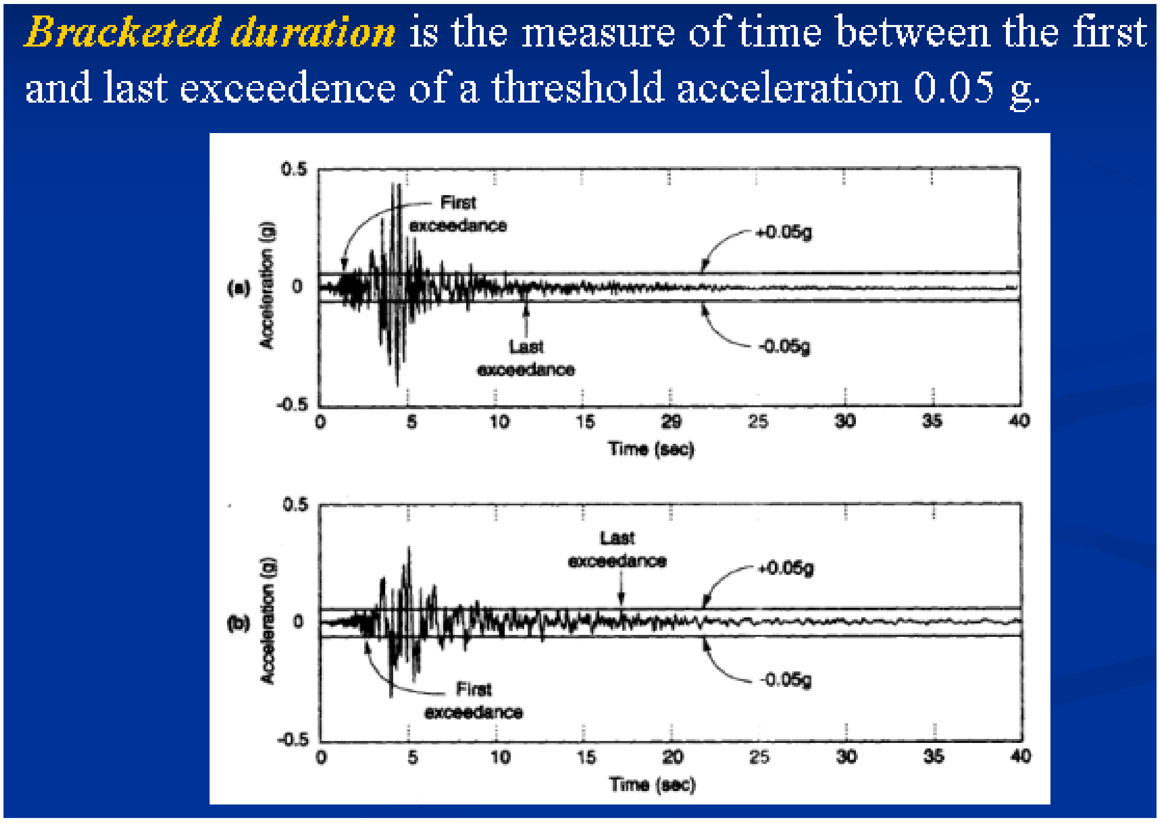
McGuire (1978) proposed a method to estimate the ratio V_{max}/a_{max} accounting for the magnitude and frequency dependencies of the ratio.

Estimation of Response Spectrum

Design spectra were developed by scaling standard spectral shapes by some ground motion parameters, usually the PHA. Earthquake magnitude has effect on spectral shape, especially in the long-period range. Relationships based on regression analyses (e.g., Boore et al. 1993) have been proposed to estimate response spectrum accounting for magnitude dependence.

Estimation of Duration

The duration of strong motion increases with earthquake magnitude. Durations based on absolute acceleration levels, such as the bracketed duration, decrease with distance. Durations based on relative acceleration levels (relative to the peak) increase with distance. For engineering purposes, the bracketed duration appears to provide the most reasonable indication of the influence of duration on the potential damage.



Estimation of rms Acceleration

Hanks and McGuire (1981):

$$a_{\max} = 0.119\frac{\sqrt{f_{\max}/f_c}}{R} \tag{24}$$

Kavazanjian et al. (1985):

$$a_{\max} = 0.472 + 0.268M_w + 0.129\log\left(\frac{0.966}{R^2} + \frac{0.255}{R}\right) - 0.1167R \tag{25}$$

Estimation of Arias Intensity

Campbell and Duke (1974):

$$I_a(m/s) = 313 \frac{e^{M_s(0.33M_s - 1.47)}}{R^{3.79}} S \tag{26}$$

Wilson (1993):

$$\log I_a(m/s) = M_w - 2\log R - kR - 0.3990 + 0.365(1-P) \tag{27}$$

Estimation of Acceleration and Velocity Spectrum Intensities

Von Thun et al. (1988) proposed charts showing the attenuation relationships for acceleration and velocity spectrum intensities.

Spatial Variability of Ground Motions

For structures such as bridges and pipelines that extend over considerable distances, different ground motions may occur under different parts of the structure. In such cases the local spatial variation (or incoherence) of the ground motion may exert an important influence on the response of the structure.

Causes:

Wave passage effect (inclined wavefront)

Extended source effect (More than one fault ruptures)

Ray-path effects (Inhomogeneous soils along the wave travel path)

Similarity Between Two Ground Motions

The similarity between ground motions at different locations can be described in the time domain or frequency domain. Consider two locations j and k at which accelerograms $a_j(t)$ and $a_k(t)$ are recorded. In the time domain, the similarity can be described by the cross covariance

$$C_{jk}(\tau) = \sum_{i=1}^{N} a_j(t_i) a_k(t_i + \tau) \tag{28}$$

Where τ is a time increment and N is the number of time samples. If the covariance of an accelerogram analyzed against itself, the autocovariance ($C_{jj}(\tau)$ or $C_{kk}(\tau)$) is obtained.

In the frequency domain, the similarity can be described by the coherency

$$\gamma_{jk} = \frac{S_{jk}(\omega)}{\sqrt{S_{jj}(\omega) S_{kk}(\omega)}} \tag{29}$$

Where the smoothed cross spectrum, $S_{jk}(\omega)$, is the Fourier transform of the cross-covariance and the autospectra, $S_{jj}(\omega)$ and $S_{kk}(\omega)$, are the Fourier transforms of the autocovariances, $C_{jj}(\tau)$ or $C_{kk}(\tau)$. The coherency describe the degree of positive or negative correlation between the amplitudes and phase angles of two time histories at each of their component frequencies. Ground motions recorded by dense arrays show that coherency decreases with distance between measuring locations and with increasing frequency.