## **Explanation of Inputs for 2DOF Program**

The 2DOF program uses modal analysis to solve for the response of the 2-story shear frame (shown below).



Note: the mass is the same for each floor.

Using Modal Analysis, we can rewrite the original coupled matrix equation of motion as a set of un-coupled equations.

$$\ddot{\mathbf{q}}_i + 2\zeta \boldsymbol{\omega} \dot{\mathbf{q}}_i + \boldsymbol{\omega}_i^2 \mathbf{q}_i = -\frac{\mathbf{L}_i}{\mathbf{M}_i} \ddot{\mathbf{u}}_g, \ i = 1, 2, ..., \text{NDOF}$$

with initial conditions of  $d_i(t=0) = d_{i_0}$  and  $v_i(t=0) = v_{i_0}$ 

We can solve each one separately (as a SDOF system), and compute histories of  $q_i$  and their time derivatives. To compute the system response, plug the q vector back into  $\mathbf{u} = \mathbf{\Phi}\mathbf{q}$  and get the u vector (and the same for the time derivatives to get velocity and acceleration).

The beauty here is that there is no matrix operations involved, since the matrix equation of motion has become a set of un-coupled equation, each including only one generalized coordinate  $q_n$ .

To run the program, you must specify values for the natural frequency, modal damping ratio, and mode shape for both modes.

The program sets the modal mass to 1 and then calculates the modal stiffness using the equation:

$$\mathbf{K}_{i} = \mathbf{M}_{i}\boldsymbol{\omega}_{i}^{2} = \mathbf{1} \cdot \boldsymbol{\omega}_{i}^{2} = \boldsymbol{\omega}_{i}^{2}.$$

Next, the program calculates a Damping (  $C_{\rm i}$  ) using the equation:

$$C_i = 2 \zeta_i \omega_i M_i$$

The modal participation factors are calculated using the equation:

$$\frac{L_{i}}{M_{i}} = \frac{\sum_{j=1}^{2} m_{j} \varphi_{ji}}{\sum_{j=1}^{2} m_{j} \varphi_{ji}^{2}}$$

However, since the mass is the same for both floors, this equation reduces to:

$$\frac{L_{i}}{M_{i}} = \frac{m\sum_{j=1}^{2} \Phi_{ji}}{m\sum_{j=1}^{2} \Phi_{ji}^{2}} = \frac{\sum_{j=1}^{2} \Phi_{ji}}{\sum_{j=1}^{2} \Phi_{ji}^{2}}$$