Earthquake Engineering Questions
DRAFT November 2011

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Note: Always include *Units* in your answer where applicable

Single-Degree-of-Freedom (SDOF) and Response Spectrum

Convert $m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_{ground}$ to an equation in terms of natural frequency (ω) and damping ratio (ζ), instead of m, c, and k.
Answer: $\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = -\ddot{u}_{ground}$

– A SDOF can be described by m, k, and c and also by frequency and damping ratio. What is the relationship between these parameters? Write the SDOF in terms of frequency and damping.

– In a SDOF idealization, m = 1 kg, find k (mention units) for a natural frequency of 0.5 Hz.

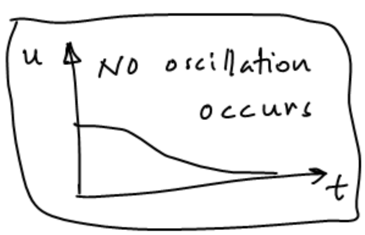
– In a SDOF idealization, m = 2.5 kg, find k (mention units) for a natural frequency of 2 Hz.

– If the weight of a SDOF structure is 16 kips and k = 48 kips/in., calculate the natural frequency in Hz (show all steps).

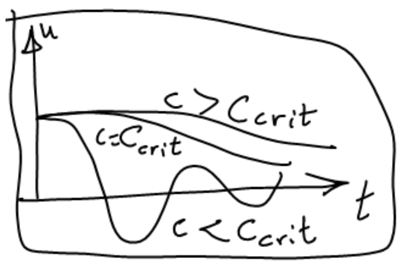
– Write the SDOF equation in terms of natural frequency and damping ratio.
Answer: $\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = -\ddot{u}_{ground}$

– Define the damping ratio in terms of c and c_{crit}.
Answer: $\zeta = c/c_{crit}$

– How does a system respond if damped above c_{crit}.
Answer: No oscillation about zero displacement occurs

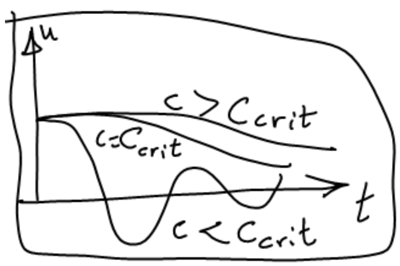


– Draw a sketch showing free vibration with damping below, at, and above c_{crit}.
Answer:



– For a SDOF, how do we calculate frequency in Hz using k and m?
  Answer: ω *= sqrt(k/m) radians/sec and f =* $\omega/2\pi$ *in Hz*

– What is critical damping (draw a sketch)?
Answer: It is the least damping that prevents oscillation



– What is $c_{critical}$ in your own words (draw a sketch as well)?

– If the weight of a SDOF structure is 18 kips and k = 38.58 kips/in., calculate the natural frequency in Hz (show all steps).

– What can you learn from the free vibration phase of a SDOF?
Answer: Natural period and damping ratio

– When does a beating-type response occur in a SDOF?
  Answer: During harmonic excitation, with the excitation frequency close to the natural frequency of the structure. The closer the frequencies, the longer the amplitude build-up/reduction phases of the system response.

– For the SDOF equation $m\ddot{u} + ku = 0$, include a viscous damping term with a damping coefficient of 5%.
  Answer: $m\ddot{u} + 0.05(c_{crit})\dot{u} + ku = m\ddot{u} + 0.05(2sqrt(km))\dot{u} + ku = 0$

– In a SDOF idealization, given k = 400 N/m, find m (mention units) for a natural period of 0.5 seconds.

– If the weight of a SDOF structure is 20 kips and k = 50 kips/in, calculate the natural period (show all steps).

- If the weight of a SDOF structure is 20 kips and k = 40 kips/in., calculate the natural frequency in Hz (show all steps).

- Show that $\frac{c}{m} = 2\xi\omega$

- Draw a labeled sketch of a damped SDOF shear building, showing the earthquake displacement and the relative displacement.

- Write a SDOF equation with damping and base earthquake excitation.

- Write equations for SDOF natural frequency in radians/sec, and natural period in seconds.

- Write the equation for critical damping in terms of m and k.

- What is a typical damping ratio for a Structure in %?

- What is a typical damping ratio for a Structure, as a number in the SDOF equation?
- Define Hz (to illustrate, draw a sketch of free vibration).

- How can you measure (conceptually) relative displacement of a SDOF shear structure using accelerometers ?

Answer:
Integrate (twice) roof acceleration to get roof displacement.
Integrate (twice) ground acceleration to get ground displacement
Subtract roof displacement from ground displacement

- Question:
  a) A viscous damping coefficient ***c*** is added in the SDOF equation, although we know that viscosity is not necessarily the dominant damping mechanism in structures. Mention a typical actual damping mechanism in structures.
  b) Why do we resort to c in our analyses?

- How do you estimate peak ground acceleration from the ***A*** response spectrum?

- For a SDOF of T = 2 sec and 2% viscous damping, D = 8 inches (in a particular spectrum). Write expressions for V and A for this SDOF (include units, and change A to g units please). Note that D is the same as S_d or SD and similarly V is S_v or PSV and A is S_a or PSA.

- During free vibration, how will a SDOF behave (draw a time history sketch)? What can be learned from the free vibration phase (mention two things please)?

- How does a SDOF system behave when it is free to vibrate? Answer: *System will vibrate at its natural frequency and the cyclic response will decay according to the system damping (from the rate of decay, we can find the viscous damping ratio).*

- For a SDOF of T = 2.2 sec. and 2% viscous damping, V = 45 in/sec. Find D and A for this SDOF (include units, and change A to g units).

- Given the weight (W) of a SDOF structure to be 7000 kN and its stiffness (k) to be 250 MN/m, find the peak relative displacement, pseudo velocity, pseudo acceleration, and peak ground acceleration (PGA) from the El Centro S00°E response spectrum below for 2% (or 10%) damping.

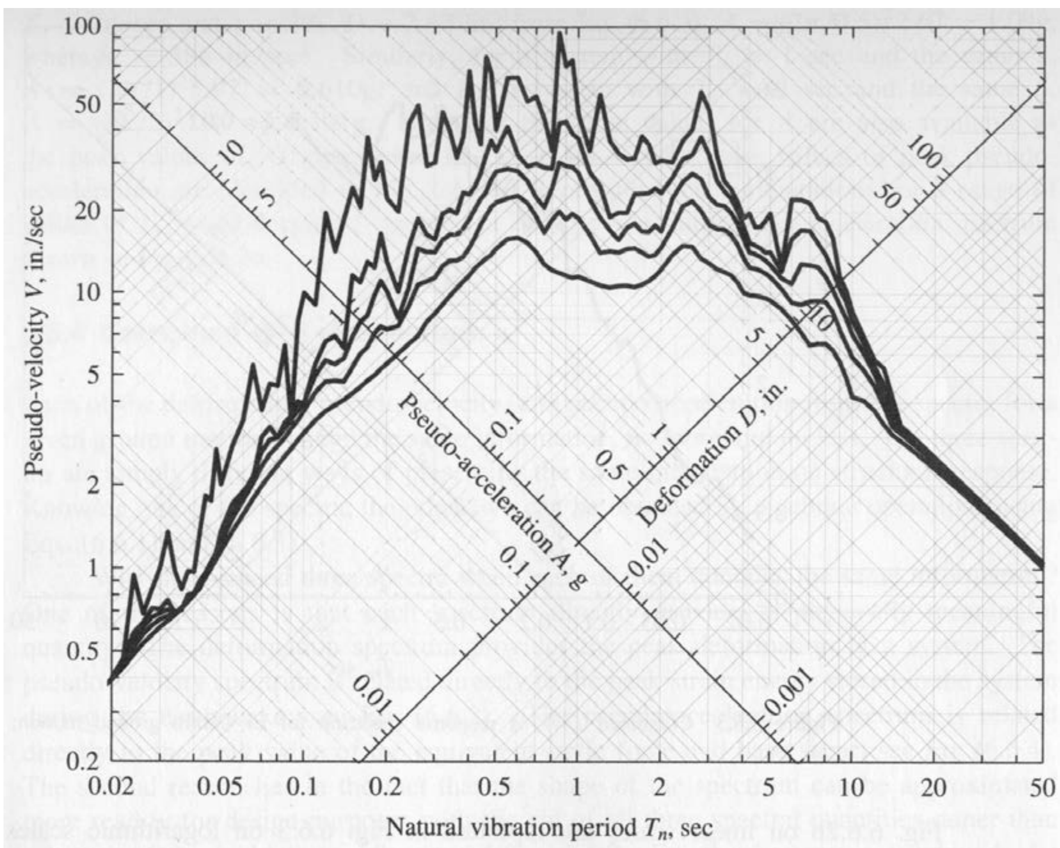


Figure 6.6.4 Combined *D–V–A* response spectrum for El Centro ground motion; ζ = 0, 2, 5, 10, and 20%.

Question:

a) What is the period of the SDOF (2% damping) that experiences maximum relative displacement during the El Centro event?
b) What is the corresponding ***V*** and ***A*** (don't forget units)?

Question:

b) What is the period of the SDOF (2% damping) that experiences maximum pseudo velocity during the El Centro event?
c) What is the corresponding ***D*** and ***A*** as observed from the response spectrum (don't forget units)?
d) Do the values of ***D***, ***V***, and ***A*** from the Spectrum figure match the mathematical relationship between ***D***, ***V***, ***A*** (show with a simple calculation)?

Question:

a. What is S_d, PSV, PSA (don't forget units) for a structure of 1.0 second natural period and 10% damping (from Figure above)?
b. Which of the curves above probably most closely looks like the input ground acceleration spectrum?
c. Estimate the peak ground acceleration from the Spectrum figure above (mark it on the sketch).
d. Estimate the peak ground displacement from the figure above.
e. Find the highest peak relative displacement (of a SDOF structure) of 2% damping, and the period at which this occurs.

– For a given SDOF structure subjected to an earthquake excitation, V = 30 in/sec and A = 0.244 g. Find D and the natural frequency in Hz.

– What is the deformation response spectrum?

– What is Pseudo acceleration spectrum. Why is it Pseudo?

– When is Pseudo acceleration an excellent estimator of absolute acceleration (with reference to period and damping)?

– What is the difference between a design spectrum and a response spectrum?

– Derive a relation showing that A is also the spectrum of actual total (absolute) acceleration for zero viscous damping.

– Why does D always start at zero for T =0 seconds?

– How do we find the peak ground displacement from a response spectrum D, and why?

– How do we find the peak ground acceleration from the pseudo spectrum A, and why?

– How do we find the peak ground displacement from a response spectrum D, and why?

– How do we find the peak ground acceleration from the pseudo spectrum A, and why?

- Draw a sketch of a D earthquake response spectrum clearly showing the value of D as $T \rightarrow 0$ and as $T \rightarrow \infty$.

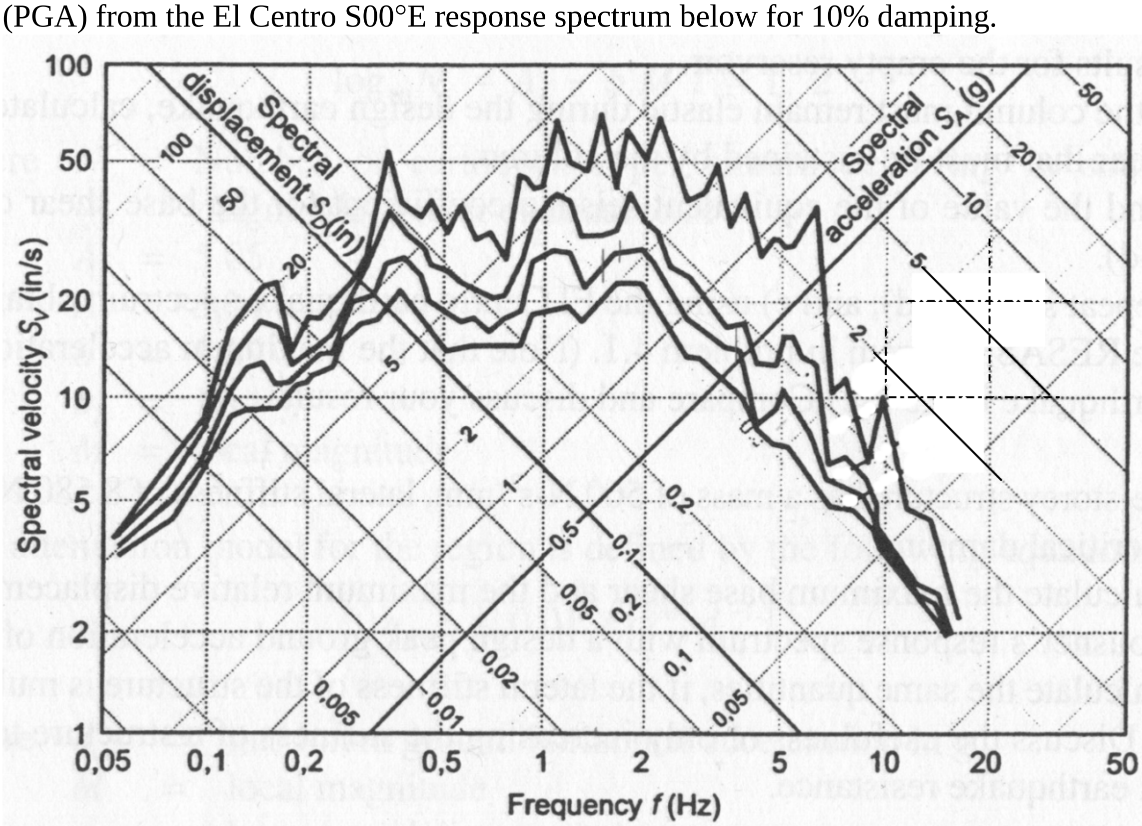
- What do we use (what do we start with as our "input") to develop a response spectrum (please be very specific)? With this input, how is the response spectrum developed?

- Sketch ***D***, ***V***, and ***A*** each on a graph showing the main difference in shape of these curves.

- How do we construct a Deformation Response Spectrum? Answer: *For a given earthquake record using the SDOF response, find the relative displacement time history for a structure with a certain damping and certain frequency. Read the max value from the graph (this value is known as* $\boldsymbol{S_d}$ *or* ***D****). Repeat process for other structures with different frequencies* ***but same damping****. Take the absolute of all the max. displacements (*$\boldsymbol{S_d}$*'s), and plot these with respect to the Structure's Natural Period. Repeat for other values of damping.*

- Give an example for use of pseudo acceleration response? Answer: *From the pseudo acceleration, the maximum force can be obtained by multiplying the mass times the maximum* S_a*.*

Given the weight (W) of a SDOF structure is 7000 kN and its stiffness (k) is 250 MN/m, find the peak relative displacement, pseudo velocity, pseudo acceleration, and peak ground acceleration (PGA) from the El Centro S00°E response spectrum below for 10% damping.



Combined D-V-A response spectrum for El Centro ground motion; ζ = 0, 2, 5, and 10%

– How do you estimate peak ground acceleration from the A response spectrum?

– For a SDOF of T = 2 sec. and 2% viscous damping, V = 30 in/sec. Find D and A for this SDOF (include units, and change A to g units please).

– Fill in the blanks: Logarithmic decrement is estimated from the ________ vibration response phase.

– Fill in the blanks with a complete definition (or draw a labeled sketch):
  If I have a S_d figure, I know the _______________________.
  Pseudo acceleration spectrum is called pseudo because ________________.

– Why is it possible to sketch S_d, PSA, PSV on the same 4-way plot figure?

– What do we use (what do we start with as our "input") to develop a response spectrum (please be very specific)? With this input, how is a response spectrum developed?

– What is the parameter we obtain from a displacement response spectrum ***D***, and why is this particular parameter of importance?

– Why is the response spectrum useful for analysis of a SDOF system?

– Why is a single response spectrum inadequate for design of a particular structure under consideration, and what do we do instead (following the logic of use of spectra)?

– How do we develop a design spectrum?

– What is the difference between a design spectrum and a response spectrum?

– Why are ***V*** and ***A*** known as "Pseudo" spectra?

– Design Spectrum Question (draw sketches as much as possible)
  a) If a fairly rigid structure is to be designed, the engineer decided to pay close attention to a smaller nearby active fault. Why?
  b) The engineer thought that this fault is not as relevant for a tall high rise building in the same area. Why?
  c) If it is decided that this nearby fault is the main concern, how would we go about developing a design spectrum for the site of interest?
  d) If there is also a distant active very large active fault, how would we develop a design spectrum for the area near the smaller fault?

– Referring to the Figure below:

a) How many earthquake records were used to derive the shown response spectrum.
b) What is the peak ground acceleration (also clearly show how to read this value from the figure).

c) What is the highest response peak acceleration ***A*** for the ζ= 2% damping case (also mark on the figure).
d) Referring to (c) above, what is the natural period of the corresponding SDOF system.
e) Referring to (d) above, what is the corresponding peak relative displacement ***D***.
f) Using a simple sketch, display the concept of relative displacement for earthquake engineering applications.
g) Referring to (d) above, what is the natural frequency in Hz of this SDOF.
h) Referring to (g) above, draw a sketch quantitatively showing the free vibration characteristics.

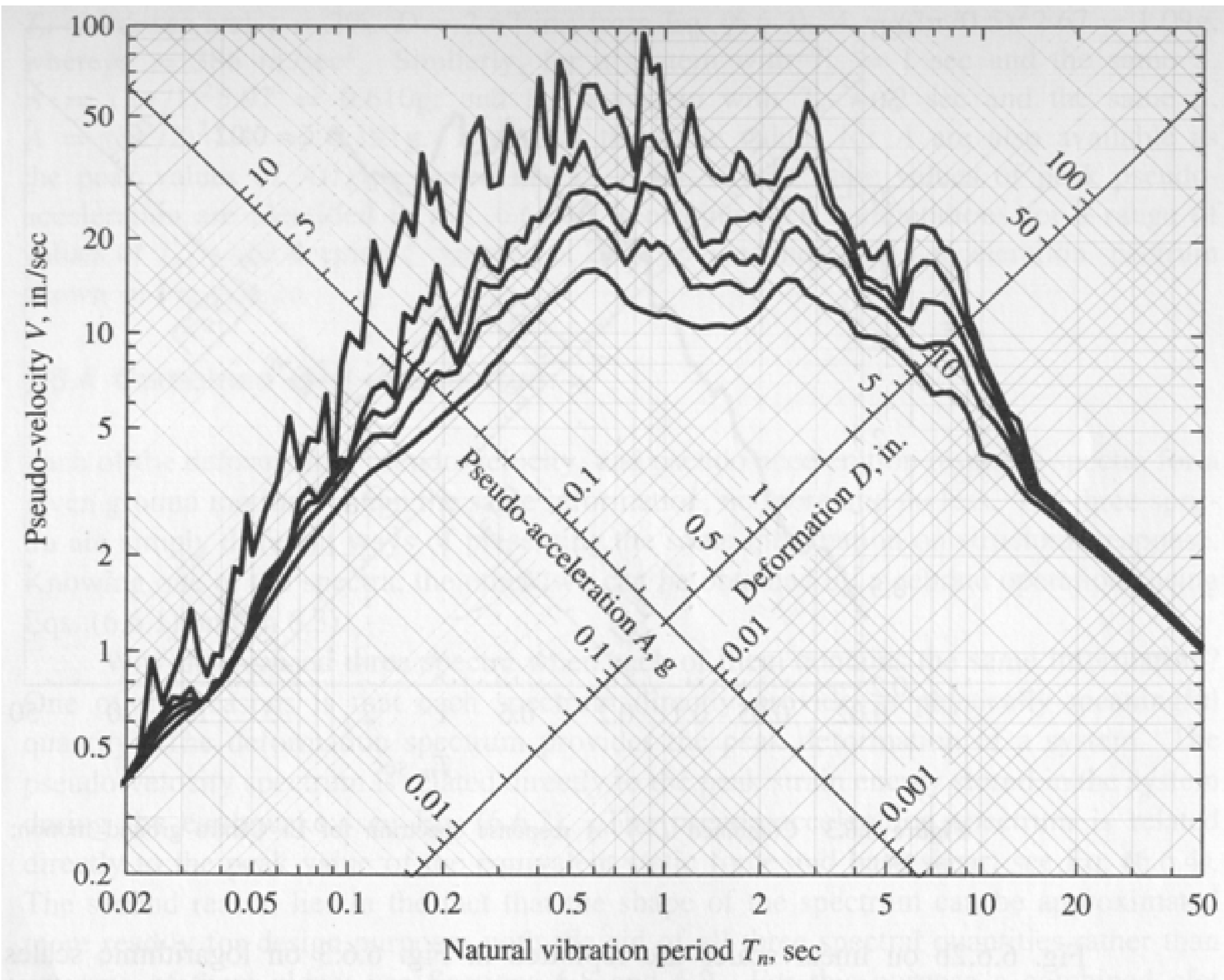


Figure 6.6.4 Combined *D–V–A* response spectrum for El Centro ground motion; $\zeta =$ 0, 2, 5, 10, and 20%.

- In SI units, if mass **m** is in **kg**, what should be the unit of stiffness **k** when calculating the natural frequency **ω** (show your calculation).

- A damping ratio $\zeta = 70\%$ denotes a level of damping that is (choose one of the responses below):

a) higher than the critical damping

b) lower than the critical damping

Answer: Lower since at critical damping, $\zeta = 100\ \%$

Multi-Degree-of-Freedom Systems (MDOF) and Response Spectrum

– Set up in matrix form the equations of motion for a 2x2 system (a 2 story shear building). Draw a sketch clearly showing the employed coordinate system with the degree of freedom for each floor.

– Draw a neat sketch showing how viscous damping changes with frequency in **c** = a_0 **m**.

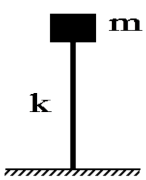
– Draw a neat sketch showing how viscous damping changes with frequency in **c** = a_1 **k**.

– If **c** = a_0 **m** + a_1 **k**, what would you typically do to define a_0 and a_1 (be specific)?

– What do you usually attempt to do in defining a_0 and a_1?

– True or false: Mass proportional damping is proportional to the inverse of frequency.

– The mass (m) supported on a bending beam is idealized as a SDOF system with $k = \frac{3EI}{L^3}$.



How was this value of k arrived at (no calculations needed, only state the approach)?

– A three story shear structure is represented by (below, $m_1 = 175{,}000$ is at roof level, m in kg, and k in MN/m):

$$[\boldsymbol{m}] = \begin{bmatrix} 175{,}000 & 0 & 0 \\ 0 & 263{,}000 & 0 \\ 0 & 0 & 350{,}000 \end{bmatrix} \qquad [\boldsymbol{k}] = \begin{bmatrix} 105 & -105 & 0 \\ -105 & 315 & -210 \\ 0 & -210 & 525 \end{bmatrix}$$

$$[\boldsymbol{\Phi}_1] = \begin{bmatrix} 1.000 \\ 0.644 \\ 0.300 \end{bmatrix} \qquad [\boldsymbol{\Phi}_2] = \begin{bmatrix} 1.000 \\ -0.601 \\ -0.676 \end{bmatrix} \qquad [\boldsymbol{\Phi}_3] = \begin{bmatrix} 1.000 \\ -2.570 \\ 2.470 \end{bmatrix}$$

and $\omega_1 = 14.5\, rad/s \quad \omega_2 = 31.1\,\mathrm{rad/s} \quad \omega_3 = 46.1\, rad/s$

a) Derive the modal participation factors.
b) Using the El Centro response spectrum (5% damping):
  b1) Estimate maximum relative roof displacement.
  b2) Estimate maximum modal floor forces (for each mode, for each floor).
  b3) Estimate maximum modal base shear (for each mode).
  b4) Estimate maximum base shear.

b5) Estimate maximum shear in columns above 1st floor.

– Using the response spectrum approach for a MDOF system, how do you estimate the maximum for any response parameter?

– For the 2-story (2 DOF) system below (mass and stiffness are given in normalized form ($kips\text{-}s^2/in$, and kips/inch)

$$\varphi_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \varphi_1 = \begin{bmatrix} 0.5 \\ 1.0 \end{bmatrix}$$

$$M = 0.010\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad K = 40.0\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

a) Show that the modes are orthogonal.
b) Find the natural frequencies.
c) Derive the uncoupled modal (generalized coordinates) equations (with 2% modal damping), including earthquake excitation a_g.
d) Using the El Centro 1940 Response Spectrum.
  d1) Find maximum floor displacements due to mode 1.
  d2) Find maximum lateral base force due to mode 1.
e) For this structure, use the UBC 1997 Code (or other) to find Design base shear and vertical distribution of lateral force.
  Assume:
  Story height h = 9 ft
  Location: Zone 4
  Soil Profile: Predominantly dense 300 ft in depth.
  Type of Structure: Steel moment resisting frame (Ordinary).
  Occupancy Category: Shelter for Emergency vehicles.
Use normalized masses above for calculation of weight of structure (in M above, m=1.0 is at roof level). Please make reasonable assumptions for additional needed information if any.

– Using the modal response spectrum approach (say the El-Centro record 1940 EW direction), the peak relative displacement due to the first mode is $u_{j1} = (L_1/M_1)\, S_{d1}\, \phi_{j1}$ in which 1 denotes mode number 1, and j denotes the floor number. In this expression:
  a) How do you find S_{d1} (be very specific)?
  b) Why is (L_1/M_1) a part of the displacement expression above?
  c) Why is ϕ_{j1} a part of the expression above?
  d) Is there any approximation involved in defining the first mode relative displacement u_{j1} above? If so, explain.

- Using the spectrum approach for MDOF systems, how do you estimate the maximum of any response value? Answer: *a) Estimate of calculation of the max. of each mode alone (there is no approximation at this point), and b) Use the square root of sum of squares SRSS or similar formula.*

- If 2 modes or 3 modes are used in a modal response spectrum solution, why is the root sum square formula (for instance) used to estimate peak displacement of floor j?

- There is an approximation involved in using the root sum square formula, but it was argued that in some situations, it's not as big a problem as might appear at first sight. Why?

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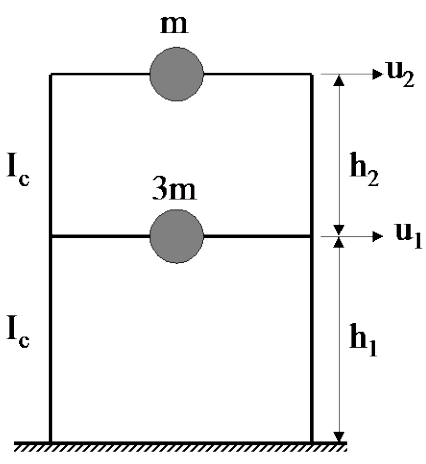
- Why is the response spectrum useful for analysis of many MDOF structures?

- Why do we define damping as a combination of mass and stiffness (MDOF systems)?

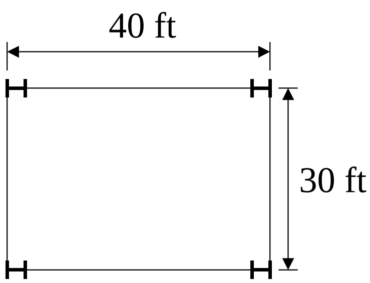
- What is the limitation of defining damping as a combination of mass and stiffness? Does this limitation apply if a modal analysis solution is undertaken?

- Sketch a 3 story idealized shear structure showing degrees of freedom u_1, u_2, and u_3 relative to the base, along with mass and stiffness coefficients m_i, k_i, i = 1, 2, 3, and base excitation u_g.
    - Write the governing matrix equation.
    - Draw free body diagram of the 2^{nd} floor.

Modal Analysis Solved Example



Elevation View



Plan View

Use:

E_{steel} = 29,000 ksi
I_c = 164.8 in^4
h_1 = 15 ft
h_2 = 12 ft

$$m = \frac{W}{g} = \left(30\frac{lb}{ft^2}\right)\left(\frac{40\,ft \times 30\,ft}{386.4\,in/s^2}\right) = 93.17\frac{lb \cdot s^2}{in} = 0.09317\frac{kip \cdot s^2}{in}$$

$$\mathrm{k_{column}} = \left(\frac{12\mathrm{EI_c}}{\mathrm{h_i^3}}\right) \Rightarrow \mathrm{k_{floor\ \mathit{i}}} = 4\left(12\frac{\mathrm{EI_c}}{\mathrm{h_i^3}}\right) = \frac{48\mathrm{EI_c}}{\mathrm{h_i^3}}$$

(a) For the two-story building shown above, define the two-degree-of-freedom free vibration matrix equation in terms of k and m. Using this matrix equation, determine the natural frequencies ω_1 and ω_2 (in terms of k and m). Using these expressions for ω_1 and ω_2, plug in numerical values and determine ω_1 and ω_2 in radians. For each natural frequency, define and sketch the corresponding mode shape.

(b) Verify that the modes are orthogonal as expected.

(c) Normalize the first mode such that $\boldsymbol{\phi}_1^{\mathbf{T}}\mathbf{m}\boldsymbol{\phi}_1 = 1.0$

(d) Use the normalized first mode (from above) to verify that $\boldsymbol{\phi}_1^{\mathbf{T}}\mathbf{k}\boldsymbol{\phi}_1 = \omega_1^2$

(e) Use the El Centro Response Spectrum and a damping ratio of 5% to estimate the maximum base shear and moment.

(f) Find $\mathrm{a_0}$ and $\mathrm{a_1}$ in $\mathbf{c} = \mathrm{a_0}\mathbf{m} + \mathrm{a_1}\mathbf{k}$ for a viscous damping of 5% in modes 1 and 2.

Calculate:

$$k_1 = \frac{48EI_c}{h_1^3} = \frac{48(29000\,ksi)(164.8\,in^4)}{(180\,in)^3} = 39.335\,kip/in$$

$$k_2 = \frac{48EI_c}{h_2^3} = \frac{48(29000\,ksi)(164.8\,in^4)}{(144\,in)^3} = 76.826\,kip/in$$

$$m = 0.09317\frac{kip \cdot s^2}{in}$$

Write the free-vibration equation of motion in matrix form:

$$\begin{bmatrix} 3m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

By substitution,

$$\begin{bmatrix} 0.27951 & 0 \\ 0 & 0.09317 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 39.335 + 76.826 & -76.826 \\ -76.826 & 76.826 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For steady state response, substitute $\ddot{\mathrm{u}} = -\omega^2 \mathrm{u}$ and rearrange to get

$$\begin{bmatrix} 116.161 - \omega^2(0.27951) & -76.826 \\ -76.826 & 76.826 - \omega^2(0.09317) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Take the determinant and set equal to zero (to find the ω^2 values that make the system singular, and thus allow for a non-trivial solution):

$$\begin{vmatrix} 116.161 - \omega^2(0.27951) & -76.826 \\ -76.826 & 76.826 - \omega^2(0.09317) \end{vmatrix} = 0$$

Let $\lambda = \omega^2$

$$\begin{vmatrix} 116.161 - \lambda(0.27951) & -76.826 \\ -76.826 & 76.826 - \lambda(0.09317) \end{vmatrix} = 0$$

Note:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

or,

$0.0260419467\ \lambda^2 - (21.4736353 + 10.8227204)\ \lambda + 8924.18499 - 5902.23428 = 0$

or,

$0.0260419467\ \lambda^2 - 32.2963557\ \lambda + 3021.95071 = 0$

Solving the above quadratic equation

Note: For the quadratic equation

$$ax^2 + bx + c = 0,$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

or

$\lambda_1 = 101.95044826669437$ and $\lambda_2 = 1138.2162747532864$ (lowest value always λ_1, then λ_2 and so forth)

Therefore (since $\lambda = \omega^2$):

$$\omega_1^2 = 101.95\ \frac{rad^2}{s^2}$$

$$\omega_2^2 = 1138\ \frac{rad^2}{s^2}$$

$$\omega_1 = 10.097\frac{rad}{s}$$

$$\omega_2 = 33.734\frac{rad}{s}$$

$$f_1 = \frac{\omega_1}{2\pi} = 1.607 Hz \qquad T_1 = \frac{1}{f_1} = 0.6223\ Sec$$

$$f_2 = \frac{\omega_2}{2\pi} = 5.369 Hz \qquad T_2 = \frac{1}{f_2} = 0.1863\ Sec$$

Determining 1^{st} Mode Shape:

Plug ω_1 = 10.097 rad/sec into $\begin{bmatrix} 116.161-\omega^2(0.27951) & -76.826 \\ -76.826 & 76.826-\omega^2(0.09317) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

$$\begin{bmatrix} 116.161-(10.097^2)(0.27951) & -76.826 \\ -76.826 & 76.826-(10.097^2)(0.09317) \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 87.665 & -76.826 \\ -76.826 & 67.327 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$87.665\phi_{11} - 76.826\phi_{21} = 0$$

let $\phi_{21} = 1$, then

$$87.665\phi_{11} - 76.826(1) = 0$$

$$\phi_{11} = \frac{76.826}{87.665} = 0.876$$

$$\Phi_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 0.876 \\ 1.000 \end{Bmatrix}$$

Determining 2nd Mode Shape:

Plug ω_2 = 33.734 rad/sec into $\begin{bmatrix} 116.161-\omega^2(0.27951) & -76.826 \\ -76.826 & 76.826-\omega^2(0.09317) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

$$\begin{bmatrix} 116.161-(33.734^2)(0.027951) & -76.826 \\ -76.826 & 76.826-(33.734^2)(0.09317) \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -201.917 & -76.826 \\ -76.826 & -29.200 \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

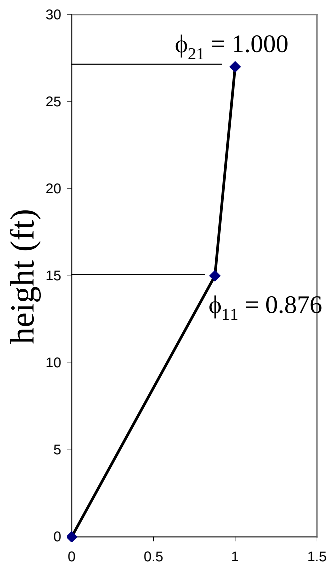
$$-201.917\phi_{12} - 76.826\phi_{22} = 0$$

let $\phi_{22} = 1$, then

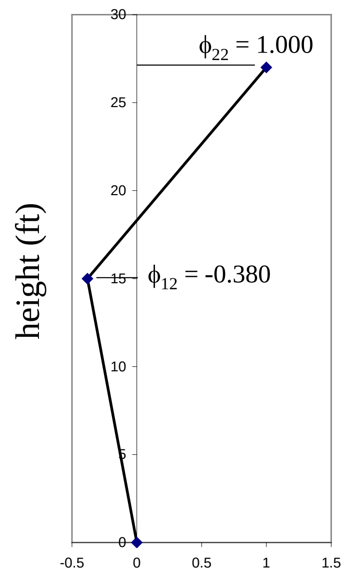
$$-201.917\phi_{12} - 76.826(1) = 0$$

$$\phi_{12} = \frac{76.826}{-201.917} = -0.380$$

$$\boldsymbol{\phi}_2 = \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} -0.380 \\ 1.000 \end{Bmatrix}$$



1st Mode Shape



2nd Mode Shape

(b) Verify that the modes are orthogonal as expected.

To verify that the modes are orthogonal, need to show that $\boldsymbol{\phi}_i^T \mathbf{m} \boldsymbol{\phi}_j = \boldsymbol{\phi}_i^T \mathbf{k} \boldsymbol{\phi}_j = 0$, for i≠j

$$\boldsymbol{\phi}_1^T \mathbf{m} \boldsymbol{\phi}_2 = \begin{bmatrix} 0.876 & 1 \end{bmatrix} \begin{bmatrix} 0.27951 & 0 \\ 0 & 0.09317 \end{bmatrix} \begin{Bmatrix} -0.380 \\ 1 \end{Bmatrix} = \begin{bmatrix} 0.024485 & 9.317x10^{-2} \end{bmatrix} \begin{Bmatrix} -0.380 \\ 1 \end{Bmatrix} = 0$$

$$\boldsymbol{\phi}_1^T \mathbf{k} \boldsymbol{\phi}_2 = \begin{bmatrix} 0.876 & 1 \end{bmatrix} \begin{bmatrix} 116.161 & -76.826 \\ -76.826 & 76.826 \end{bmatrix} \begin{Bmatrix} -0.380 \\ 1 \end{Bmatrix} = \begin{bmatrix} 24.9310 & 9.5264 \end{bmatrix} \begin{Bmatrix} -0.380 \\ 1 \end{Bmatrix} = 0$$

Can also show $\boldsymbol{\phi}_2^T \mathbf{m} \boldsymbol{\phi}_1 = \boldsymbol{\phi}_2^T \mathbf{k} \boldsymbol{\phi}_1 = 0$

(c) Normalize the first mode such that $\boldsymbol{\phi}_1^{\mathbf{T}} \mathbf{m} \boldsymbol{\phi}_1 = 1.0$

$$\boldsymbol{\phi}_1^T \mathbf{m} \boldsymbol{\phi}_1 = \begin{bmatrix} 0.876 & 1 \end{bmatrix} \begin{bmatrix} 0.27951 & 0 \\ 0 & 0.09317 \end{bmatrix} \begin{Bmatrix} 0.876 \\ 1 \end{Bmatrix} = 0.30766$$

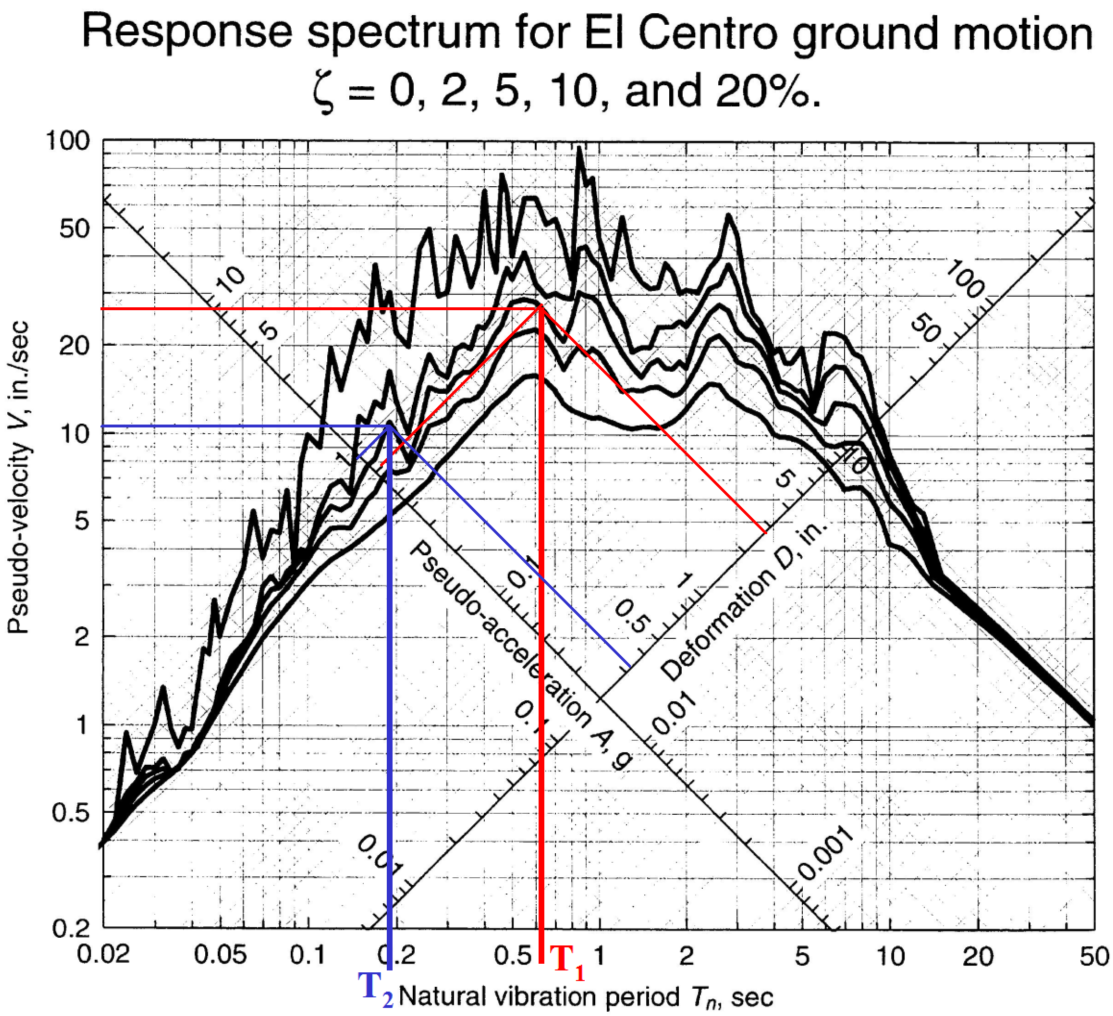
Divide $\boldsymbol{\phi}_1$ by $\sqrt{0.30766}$, therefore $\boldsymbol{\phi}_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 1.5793 \\ 1.8029 \end{Bmatrix}$

Check: $\boldsymbol{\phi}_1^T \mathbf{m} \boldsymbol{\phi}_1 = \begin{bmatrix} 1.5793 & 1.8029 \end{bmatrix} \begin{bmatrix} 0.27951 & 0 \\ 0 & 0.09317 \end{bmatrix} \begin{Bmatrix} 1.5793 \\ 1.8029 \end{Bmatrix} = 1.00$ ***ok***

(d) Use the normalized first mode (from above) to verify that $\boldsymbol{\phi}_1^{\mathbf{T}} \mathbf{k} \boldsymbol{\phi}_1 = \omega_1^2$

$$\boldsymbol{\phi}_1^T \mathbf{k} \boldsymbol{\phi}_2 = \begin{bmatrix} 1.5793 & 1.8029 \end{bmatrix} \begin{bmatrix} 116.161 & -76.826 \\ -76.826 & 76.826 \end{bmatrix} \begin{Bmatrix} 1.5793 \\ 1.8029 \end{Bmatrix} = 101.950 \frac{rad^2}{s^2} = \left(10.097 \frac{rad}{s} \right)^2 = \omega_1^2$$

(e) Use the El Centro Response Spectrum and a damping ratio of 5% to estimate the maximum base shear and moment.



From Response Spectrum:

For T_1 = 0.622 sec and ζ = 5%, D_1 = 2.8 in. and A_1 = 0.8g = 309.12 in/s^2

For T_2 = 0.0.186 sec and ζ = 5%, D_2 = 0.31 in. and A_2 = 0.9g = 347.76 in/s^2

$$f_{jn} = \frac{L_n}{M_n} A_n m_j \phi_{jn}$$

$$\frac{L_i}{M_i} = \frac{\sum_{j=1}^{2} m_j \phi_{ji}}{\sum_{j=1}^{2} m_j \phi_{ji}^2}$$

$$\frac{L_1}{M_1} = \frac{\sum_{j=1}^{2} m_j \phi_{j1}}{\sum_{j=1}^{2} m_j \phi_{j1}^2} = \frac{m_1\phi_{11} + m_2\phi_{21}}{m_1\phi_{11}^2 + m_2\phi_{21}^2} = \frac{(0.27951)(0.876) + (0.09317)(1)}{(0.27951)(0.876)^2 + (0.09317)(1)^2} = 1.0987$$

$$\frac{L_2}{M_2} = \frac{\sum_{j=1}^{2} m_j \phi_{j2}}{\sum_{j=1}^{2} m_j \phi_{j2}^2} = \frac{m_1\phi_{12} + m_2\phi_{22}}{m_1\phi_{12}^2 + m_2\phi_{22}^2} = \frac{(0.27951)(-0.380) + (0.09317)(1)}{(0.27951)(-0.380)^2 + (0.09317)(1)^2} = -0.0977$$

$$f_{11} = \frac{L_1}{M_1} A_1 m_1 \phi_{11} = (1.0987)\left(309.12\,\frac{in}{s^2}\right)\left(0.27951\,\frac{kip \cdot s^2}{in}\right)(0.876) = 83.159\ kips$$

$$f_{12} = \frac{L_2}{M_2} A_2 m_1 \phi_{12} = (-0.0977)\left(347.76\,\frac{in}{s^2}\right)\left(0.27951\,\frac{kip \cdot s^2}{in}\right)(-0.380) = 3.609\ kips$$

$$f_{21} = \frac{L_1}{M_1} A_1 m_2 \phi_{21} = (1.0987)\left(309.12\,\frac{in}{s^2}\right)\left(0.09317\,\frac{kip \cdot s^2}{in}\right)(1) = 31.643\ kips$$

$$f_{22} = \frac{L_2}{M_2} A_2 m_2 \phi_{22} = (-0.0977)\left(347.76\,\frac{in}{s^2}\right)\left(0.09317\,\frac{kip \cdot s^2}{in}\right)(1) = -3.166\ kips$$

Base Shear

$$V_{0n} = \sum_{j=1}^{N} f_{jn}$$

$$V_{01} = \sum_{j=1}^{2} f_{j1} = f_{11} + f_{21} = 83.159\ kips + 31.643\ kips = 114.80\ kips$$

$$V_{02} = \sum_{j=1}^{2} f_{j2} = f_{12} + f_{22} = 3.609\ kips - 3.166\ kips = 0.443\ kips$$

Calculate Maximum Base Shear, $V_{0\,max}$

$$V_{0\max} = \sqrt{(V_{01})^2 + (V_{02})^2} = \sqrt{(114.80\ kips)^2 + (0.443\ kips)^2} = 114.80\ kips$$

Base Moment

$$M_{0n} = \sum_{j=1}^{N} f_{jn} d_j$$

$$M_{01} = \sum_{j=1}^{2} f_{j1} d_j = f_{11}d_1 + f_{21}d_2 = 83.159\ kips(180\ in) + 31.643\ kips(324\ in) = 25221.0\ kips \cdot in$$

$$M_{02} = \sum_{j=1}^{2} f_{j2} d_j = f_{12}d_1 + f_{22}d_2 = 3.609\ kips(180\ in) - 3.166\ kips(324\ in) = -376.16\ kips \cdot in$$

Calculate Maximum Base Moment, $M_{0\ max}$

$$M_{0\max} = \sqrt{(M_{01})^2 + (M_{02})^2} = \sqrt{(25221.0\ kips \cdot in)^2 + (-376.16\ kips \cdot in)^2} = 25223.8\ kips \cdot in = 2102.0\ kips \cdot ft$$

(f) Find a_0 and a_1 in $\mathbf{c} = a_0\mathbf{m} + a_1\mathbf{k}$ for a viscous damping of 5% in modes 1 and 2.

$\zeta_1 = \zeta_2 = \zeta = 0.05$

$$a_0 = \xi \frac{2\omega_i\omega_j}{\omega_i + \omega_j} = (0.05)\frac{2(10.097\ rad/s)(33.734\ rad/s)}{10.097\ rad/s + 33.734\ rad/s} = 0.777\ rad/s$$

$$a_1 = \xi \frac{2}{\omega_i + \omega_j} = (0.05)\frac{2}{10.097\ rad/s + 33.734\ rad/s} = 0.00228\ \frac{1}{rad/s}$$

$$a_0 = 0.777\ rad/s \quad \& \quad a_1 = 0.00228\ s/rad$$

Modal Analysis Procedure

Start with the original matrix equation of the system

$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{k}\,\mathbf{u} = -\,\mathbf{m}\,\mathbf{1}\,\ddot{u}_g$ (**1** is just a unit vector)

Model Analysis (Mode shapes and natural frequencies)

We seek steady-state, free-vibration ***harmonic*** response which dictates

$$\ddot{u} = -\omega^2 u$$

Substitute above (free vibration)

$$[\mathbf{k} - \omega^2 \mathbf{m}]\mathbf{u} = \mathbf{0} \qquad \textbf{Eq (*)}$$

Non trivial if Det $[\mathbf{k} - \omega^2\mathbf{m}] = 0$, or, $|\,\mathbf{k} - \omega^2\mathbf{m}\,| = 0$
(i.e., we find ω's that make the determinant equal to zero)

For 2 x 2 system the determinant equation is a quadratic equation in $\lambda = \omega^2$, which can be solved to obtain ω_1 and ω_2?

Use ω_1 in * to find $\mathbf{u}_1 \Rightarrow \phi_1$ and Use ω_2 in * to find $\mathbf{u}_2 \Rightarrow \boldsymbol{\phi}_2$ (both within a multiplier, …)? because $|\boldsymbol{k} - \omega^2\boldsymbol{m}| = 0$ *when* ω_1 or ω_2 are used, and in these 2 cases, a non zero u within a multiplier can be found.

Now mode shapes are known, so describe **u** in terms of mode shapes

$$\mathbf{u}(\mathbf{x}, t) = \boldsymbol{\Phi}(\mathbf{x})\;\mathbf{q}(t)$$

↑ ↑ ↑ ↑
space time space time

where $\boldsymbol{\Phi}$ is the modal matrix (separation of variables) composed of mode shape vectors.

Substitute in

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{k}\,\mathbf{u} = -\,\mathbf{m}\,\mathbf{1}\ddot{u}_g$$

and pre-multiply by $\boldsymbol{\Phi}^T$ to get uncoupled equations of the form:

$$\ddot{q}_i + \omega_i^2\, q_i = -\frac{L_i}{M_i}\ddot{u}_g \qquad , i= 1, 2, ndof$$

Solve each i equation independently, after adding any desired damping term $(+2\xi_i\omega_i q_i)$
to get $q_i(t)$, $\dot{q}_i(t)$ and $\ddot{q}_i(t)$
Solution is

$\mathbf{u} = \boldsymbol{\Phi}\ \boldsymbol{q}(t)$

$\ddot{\mathbf{u}} = \boldsymbol{\Phi}\ \ddot{\boldsymbol{q}}(t)$ and

$\ddot{\mathbf{u}}^{\mathbf{t}} = \ddot{\mathbf{u}} + \mathbf{1}\ddot{u}_g$ (the absolute acceleration)

where only the first few modes may be used in the modal matrix (and generalized coordinates) (e.g., first 3 or 4 modes for instance … unless the structure is quite flexible with many low frequency modes…)

Advantages of a Model Analysis Solution:

1) Do not have to solve coupled matrix equation (instead, solve uncoupled equations…)
2) Might only need to solve 3 or 4 uncoupled equations in q_i (first 3 or 4 mode shapes..)
3) Can specify damping conveniently in each mode of interest
4) Can use response spectrum procedure to get approximate solution.

– What is gained by adopting a Caughey-type damping (graduate class only)?
  a.
  b.

– Write the two equations that define modal orthogonality.

– How can the displacement of a 2 DOF's structure be described using modal analysis?
  Any shape can be described using the first two modes and some coefficients, q1(t) and q2(t.) Everything (u_1(t), u_2(t)) will be dependent on time because the structure's shape is not fixed.

– True or false: Mode shapes can be used to define any motion the structure is capable of performing.

– True or False: Mass proportional damping is proportional to the inverse of frequency.

– A 2 DOF system is represented by 2 uncoupled modal equations:
  b) Write a general expression for these equations (in terms of generalized coordinates).
  c) The structure is deformed in the shape of mode 1 and released. How does this deformed configuration trigger a response as dictated by the modal equations.
  d) Will the response remain in mode 1 throughout?

– Derive in matrix form the equations of motion for a 3x3 MDOF system (draw sketch and show all symbols used in your equations).

– Can any displaced shape **u** of a 2 DOF system be represented in terms of its modes (show using simple equation)?

– Describe the steps involved in deriving mode shapes.

– Why do we get a relationship of the form $2\,u_1 - u_2 = 0$, and not unique values for u_1 and u_2 when we're solving for a mode shape (2 DOF system)? Why is that ok anyway?

– Mention three advantages when using modal analysis to solve a MDOF system under earthquake excitation.

– If you have modes and frequencies for a 2 DOF system, write down the steps involved to solve for earthquake excitation using modal analysis.

– Why might higher modes contribute little to seismic structural response (give 2 reasons)?

– What is the term in a modal equation that shows that higher modes might contribute little to seismic response?

– What might be considered as advantages in going from a matrix equation of motion to an un-coupled modal system (give 4 reasons)?

==

– For a 3 story shear building, write a matrix equation representing floor displacements in terms of the first 2 mode shapes and generalized coordinates only. Draw a clear sketch showing your coordinate system, and modal coordinates (for the first 2 modes).

– Write an equation showing the characteristic of modal orthogonality.

– If a mode is normalized such that $\phi_n^T \mathbf{m} \phi_n = 1.0$, then $\phi_n^T \mathbf{k} \phi_n$ = _______.

– What happens to [$\mathbf{k} - \omega^2 \mathbf{m}$] if a natural frequency is substituted for ω.

– If ϕ is a mode, $x\phi$ is the same mode, where x is any nonzero number, why?

– If $\mathbf{u} = \underline{\phi}\, \mathbf{q}$ where $\underline{\phi}$ is the modal matrix, how come using $x\phi_1$ instead of ϕ_1 for instance, will have no impact on the final calculation of $\mathbf{u}$? Specifically show what compensates for that.

– Draw a sketch of the first 4 mode shapes of a shear building (show which is which).

– Draw a sketch and write an equation expressing relative displacement of a three story shear building in terms of its first mode response only.

– Write the corresponding uncoupled modal equation (with 2% viscous modal damping).

==

– Mathematically, would there be a problem if it is decided to use modes 1 and 3 to define u in a 2 mode solution (a 3 story shear building situation for instance). Explain?

==

– Describe the process of finding the mode shapes of a structure with two lumped masses (one on top of the other.) Answer: *Just like the homework; Find m and k matrix, find matrix* $\left[k - \omega_n^2 \cdot m\right]$, *find the determinant of this matrix and set it equal to zero, solve for* ω_{n1} *and* ω_{n2}, *solve for the natural frequencies, and then set the matrix times the mode vector equal to zero* $\left[k - \omega_n^2 \cdot m\right]\varnothing = 0$, *and find mode 1 and mode 2.*

– Is the normalization of the modes a required step? What are the advantages of this process? **Answer: No, this is an optional step. The advantage is that by normalizing the mode you get:** $\phi_n^T m \phi_n = 1.0$ **and** $\phi_n^T k \phi_n = \omega_n^2$

– Why do we get a relationship like this $2u_1 - u_2 = 0$, and not the values for u_1 and u_2 when we are trying to solve for the deformations in the structure? Answer: *Because we are working with a singular system of equations. The rows and the columns of the matrix are dependent of each other and this is not enough to solve for two unknowns.*

– After getting ω_1 and ω_2, does this guarantees that our system is singular? Answer: *Yes, because* ω_1 *and* ω_2 *are the two roots that solve our singular quadratic equation.*

– What modes should we use when trying to find the ones that participate most? Answer: *Use the first 2 or 3 modes. Structure has to be shaken at a high frequency to get higher modes to contribute. Earthquakes, usually, do not have that high frequency.*

– If you have a MDOF system why would you use modal analysis to solve for your unknowns? Answer: *Modal analysis uncouples the equations so that they can be treated as SDOF. The solution should be represented using all modes, but we only use 2 or 3 because they contribute the most, so for a 2 DOF system you system becomes 2 uncoupled equations.*

– Why are higher modes, more difficult to excite? Answer: *For the same ground motion we get less energy from higher modes because earthquakes with low frequencies do not excite these modes.*

– Describe the modal analysis process for a 2 DOF system.

Answer:

a) *Solve for free vibration and assume harmonic oscillation.*
b) *Perform the eigenvalue analysis (find m and k matrix, find matrix* $\left[k - \omega_n^2 \cdot m\right]$, *find the determinant of this matrix and set it equal to zero, solve for* ω_{n1} *and* ω_{n2}, *solve for the natural frequencies, and then set the matrix times the mode vector equal to zero* $\left[k - \omega_n^2 \cdot m\right]\varnothing = 0$, *and find the mode vectors 1 and 2)*
c) *Normalization is optional*

d) *Diagonalize the system by pre-multiplying by $\varnothing_n$* ***transform*** *and post-multiplying by $\varnothing_n$. By orthogonality system should become diagonal.*
e) *Get the SDOF equations and solve*
f) *Input level of damping that you want*

– What is a difference between solving for a 2DOF and a 3DOF system (using modal analysis? Answer: *A 3DOF system would be more difficult only because the eigenvalue process involves a cubic equation instead of a quadratic. All of the process is exactly the same.*

==

– Why are modes described by the word "shape"?

(a) Can a linear combination of mode shape describe any deformed shapes the structure might assume?

(b) What if the structure becomes nonlinear and accumulates permanent drifts?

(c) Can other appropriate sets of vectors (other than mode shapes) be used to represent any deformed shape of a structure? If so, give an example.
(sines and cosines or finite elements)

==

– Why is using the first mode to define the dynamic response not necessarily appropriate when a specific steady-state harmonic excitation source is acting (not a earthquake)? What is a better approach in this case?

Building Code

$V = (S_{DS} / (R/I))\ W$

not to exceed

$V = ((S_{D1} / (R/I)) / T)\ W$

In this expression which terms collectively defines the contribution of lateral acceleration as estimated by the Code?
Answer: $(S_{DS} / (R/I))$ or $((S_{D1} / (R/I)) / T)$

Why is the S_{D1} version divided by T?
Answer: The S_{D1} expression represents the part of the Code Design Spectrum that varies with (1/T)

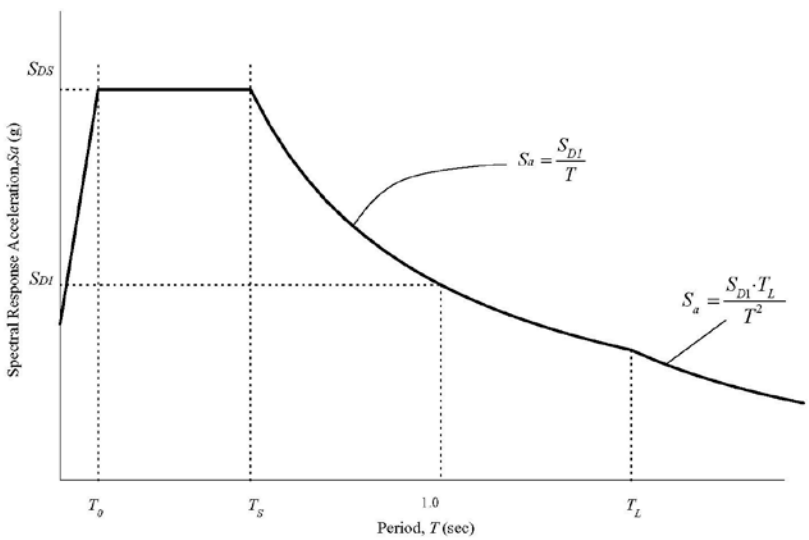


FIGURE 11.4-1 DESIGN RESPONSE SPECTRUM

What is W
Answer: It is the effective seismic weight (mass of the structure for earthquake loading calculation)

What is the shape of Figure 11.4-1 above based on:
*Answer: This figure attempts to match in a simple way (statistically) the average shape of a design spectrum that is obtained for instance by taking the average of a large number of Response Spectra (**A**). The 1/T segment captures the reduction in demand (peak acceleration) as the fundamental Period (T) of the structure increases. At TL, and sharper decrease is specified to more closely match observed earthquake response (Spectra), where Spectral acceleration is on its way to virtually disappearing for very long Period structures (ones with minimal stiffness for instance where ω is nearly zero.*

What is S_{DS} representing?
Answer: This is the Code defined Short Period Spectral acceleration, which depends on seismicity at a particular geographic location, and on local site condition.

Why is the S_{D1}/T section capped at S_{DS} as a maximum?
Answer: The shape of the design spectrum is derived from averaging response spectra of many recorded ground motions. This average generally conforms to the Response Design Spectrum of the Code, with the flat plateau of the S_{DS} portion.

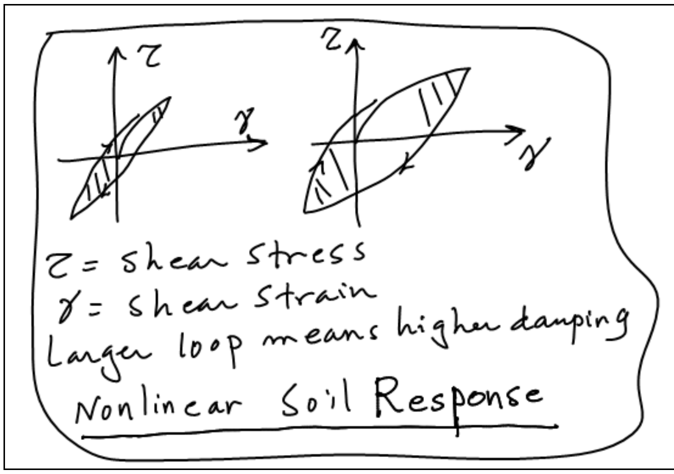
Why is R in the denominator and what is its role?
Answer: R is a factor greater than 1.0 that reduces the Code demand for structures built to be ductile, and thus more resistant to catastrophic collapse.

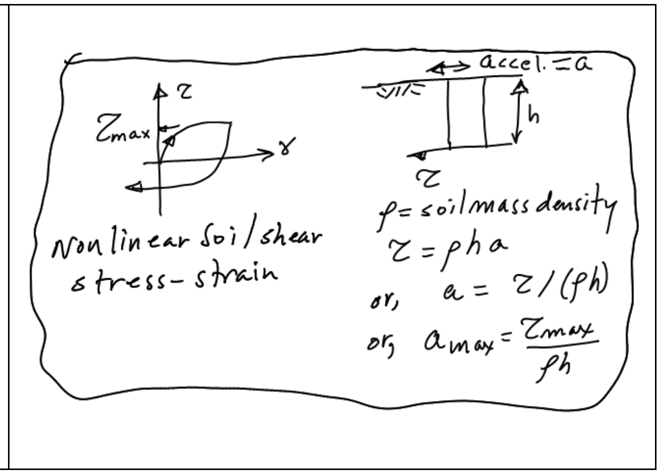
What in the anticipated main outcome in complying with the seismic code?
Answer: The aim is to save lives (prevent catastrophic collapse, such as the pancake mode of failure for instance).

As soil at the site becomes softer (weaker), S_{DS} values increase, but then might decrease. Why?
Answer: $S_{DS} = (2/3)\, S_{MS} = (2/3)\, F_a\, S_S$ *, where* S_S *is Spectral acceleration at T = 1.0 Seconds (the Short period spectral acceleration at a particular geographic location), and* F_a *is the site class adjustment factor. The coefficient* F_a *is generally larger for softer sites (e.g, site class C or D) compared to site class B for instance (stiffer), because weaker soil results in higher amplification of the incoming rock motions. However, at high levels of shaking (*$S_S = 1.0$ *or more), it decreases a bit for site class E due to expected high nonlinear behavior of this weak soil (with much larger damping and lower shear strength)*

Table 11.4-1 Site Coefficient, F_a (see Appendix 8of Code handout for site Class)

Site Class	Mapped Maximum Considered Earthquake Spectral Response Acceleration Parameter at Short Period				
	$S_S \leq 0.25$	$S_S = 0.5$	$S_S = 0.75$	$S_S = 1.0$	$S_S \geq 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	0.9
F	See Section 11.4.7 (see Appendix 9)				





For site classes C, D, and E above, why do the F_a coefficients decrease with the increase in S_S
Answer: Because of the potential for nonlinear response as the Ss value increases (which would increase the damping and reduce the shear strength.

Why are code specified V forces possibly lower than forces that a structure might experience during a seismic event (e.g., $S_{DS} = (2/3)\ S_{MS}$ *where* S_{MS} is the MCE (Maximum Considered Earthquake), 5 percent damped, spectral response acceleration at short periods adjusted for Site Class effects?
Answer: Because the code only aims to prevent catastrophic collapse (i.e., not necessarily to make a structure survive unscathed (intact) after an extremely strong seismic event).

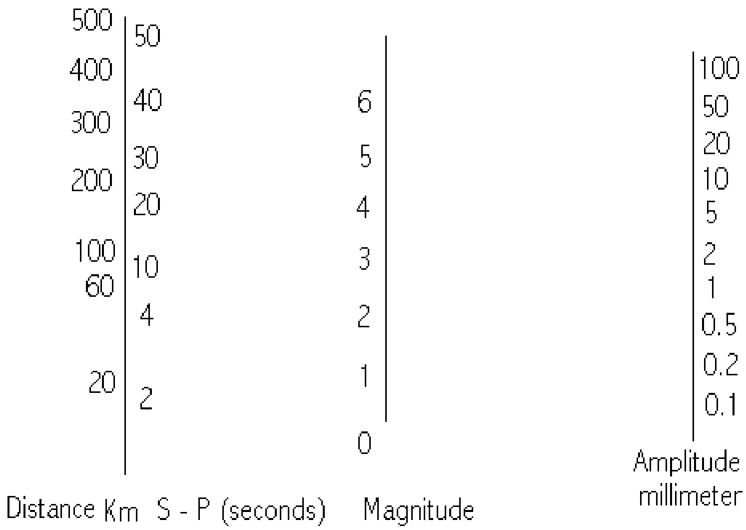
Question: Why are the F_a coefficients all equal to 1.0 for site class B (in Table 11.4-1 above).
Answer: All S_s *published values (or accessible from the website) are based on the assumption that the site is California Rock (soft Rock), which is site class B. Therefore, site adjustment coefficients* F_a *are all 1.0 for this site class (B).*

Why are the $\boldsymbol{F_a}$ values less than 1.0 for site class A
Answer: Site class A represents Stiff Hard Rock sites that are possibly encountered on the East Coast of the US (among). These rocks are significantly stiffer than California Rock (soft rock) that is represented by site class B. In view of the increased stiffness of site class A (compared to B), motion amplification will tend to be lower (for instance, the conceptual "rigid" rock scenario will result in base acceleration exactly equal to surface acceleration without any amplification). With F_a *values equal to 1.0 for Site class B, it thus makes sense that* F_a *values would be less than 1.0 for site class A.*

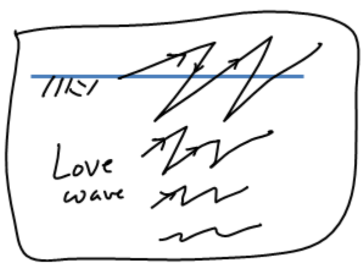
===

Seismology

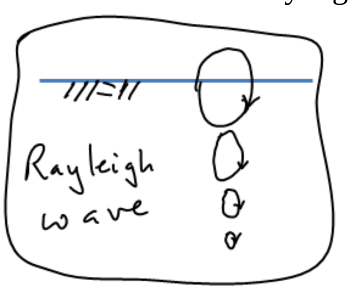
– What is the Richter magnitude for a California earthquake registering a Wood-Anderson equivalent peak amplitude of 50 mm at a distance of 300 km from the focus?



Draw a sketch of Love wave propagation (particle motion and propagation characteristics).

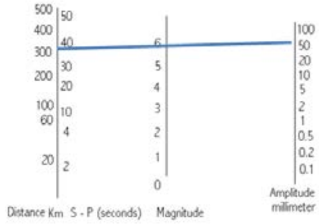


Draw a sketch of a Rayleigh wave (particle motion and propagation characteristics).



What is the Richter magnitude for a California earthquake registering a Wood-Anderson equivalent peak amplitude of 50 mm at a distance of 300 km from the focus?

Answer:

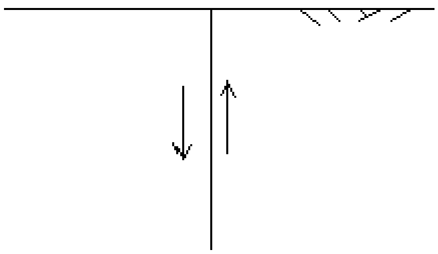


What is the mechanism behind most released seismic energy?
Answer: Shear wave energy

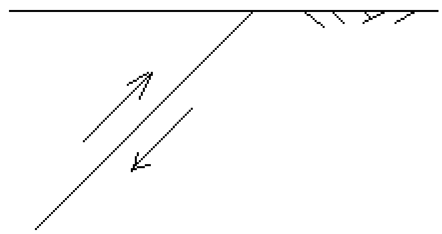
To what geologic period do the really old rocks belong?

V_p and V_s measured at a soil site are 3000 ft/sec and 812 ft/sec, respectively. The soil unit weight is 125 lb/ft^3. Calculate Young's modulus E, Shear Modulus G, and Poisson's ratio ν, for the soil.
Answer:

Describe relative fault movement and sketch focal mechanism.

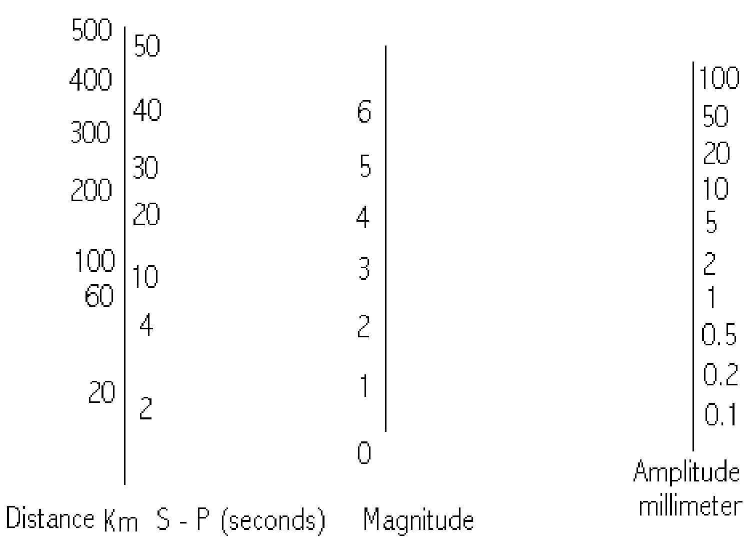


Case (a) Elevation View

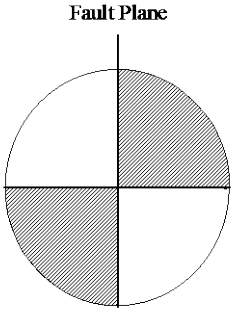


Case (b) Elevation View

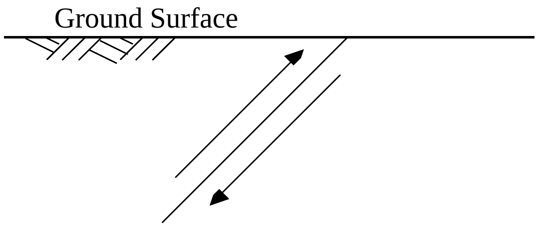
A Wood-Anderson seismograph record showed a maximum amplitude of 50 mm, and S-P arrival time was estimated to be 10 seconds. <u>However, rock **stiffness** in the affected area was **1.6 times higher** than in California</u>.
Use the Chart below to estimate a Local Magnitude for this earthquake (explain the needed modification).



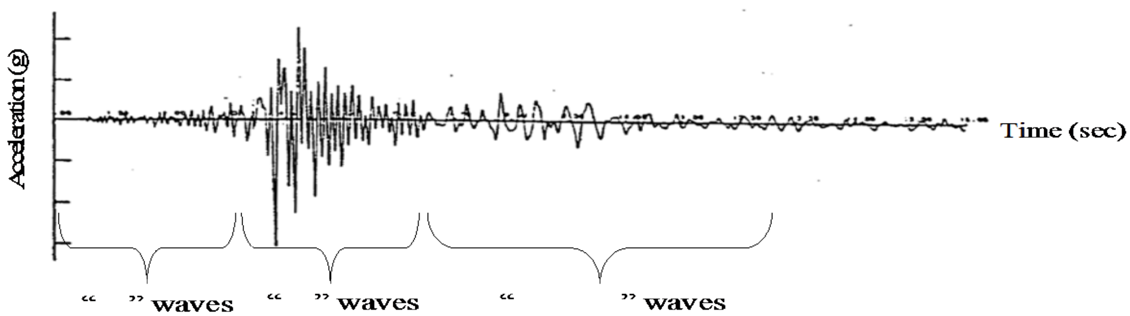
For the following focal mechanism, sketch and describe the fault movement.



Describe relative fault movement and sketch the focal mechanism.



Fill in the blanks



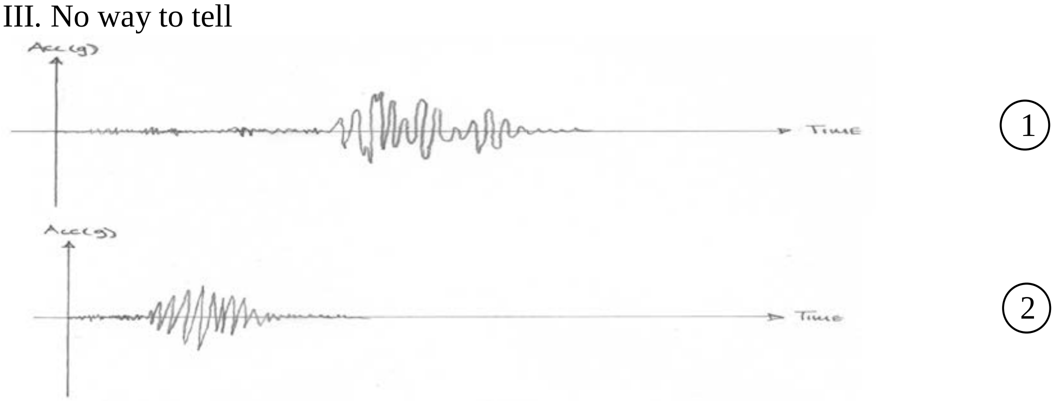
Answer: P-waves followed by S-waves, followed by surface waves

For these two synchronized earthquake records (below), choose one answer only:

I. 1 is far from earthquake source and 2 is close

II. 2 is far from earthquake source and 1 is close

III. No way to tell



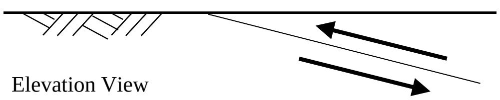
Name the two types of seismic body waves.
Answer: P-waves and S-waves

How do we locate an earthquake using the data from multiple seismographic stations (include a sketch)?

Answer: By triangulation. S-P wave arrivals dictate distance from source t (at least) three different recoding stations.

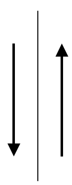
For questions (a) and (b), describe relative fault movement and sketch the focal mechanism (beach ball).

(a)



Elevation View

(b)



Plan View

List two natural disasters associated with the 1964 Alaska Earthquake.
Answer: Tsunami and major ground failures (donwslope lateral spreading)

List three types of structural (building) damage associated with earthquakes.

What is the significance of the New Madrid series of earthquakes?
Answer: Large earthquakes can occur in Mid-America and cause damage as far as New Your and Boston

Describe a notable feature of the 1985 Michoacan earthquake regarding damage in Mexico City.
Answer: Part of Mexico city is on a deep Bowl shaped layer of soft clay. This clay in the bowl acted like Jelly (with linear stress-strain response) and gradually shock back and forth building

up large shaking amplitudes at ground surface due to the incoming very low levels of shaking along the bowl boundaries.

What were main causes of damage in the 1906 San Francisco earthquake?
Answer: Fire and liquefaction in the Marina District.

What is the main cause of a near-source seismic threat to UCSD?
Answer: The Rose Canyon Fault

For a P-wave, what does the P- stand for?
Answer: P stand for "Primary"or "Pressure"

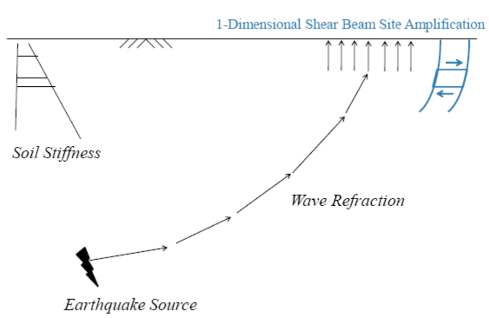
For the S-wave, what does the S- stand for?
Answer: S stands for "Secondary" or "Shear"

Which body wave (P & S) propagates faster and why?
Answer: P waves propagate faster. P wave velocity is proportional to sqrt(B+(4/3)G), while S-wave velocity is proportional to sqrt(G) only.

For a linear isotropic elastic material, stress-strain is related by moduli such as Young's modulus, Poisson's ratio, bulk modulus (B), shear modulus (G), and λ (one of Lame's constants). How many independent constants fully describe the stress-strain response of this material?
Answer: Only two independent Moduli fully describe the stress-strain behavior of a linear isotropic elastic material (actually, any two of the above).

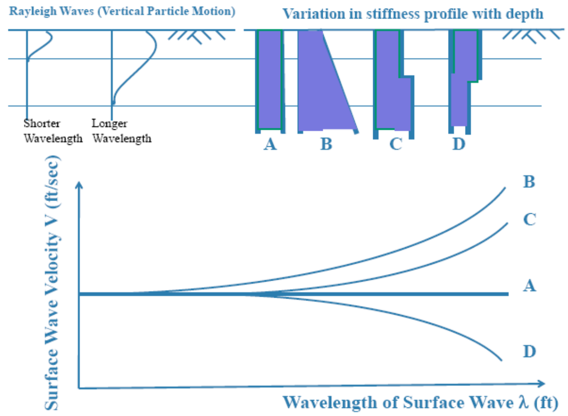
Draw a sketch that shows the reason for vertical wave propagation near ground surface
Answer:

Vertical wave incidence near ground surface



Surface waves of different wave-lengths (frequencies) may travel (near ground surface) at different velocities (wave dispersion mechanism). Why would this happen? How can this phenomenon be used to learn more about the site properties (briefly explain in a couple of sentences)?
Answer: See question below

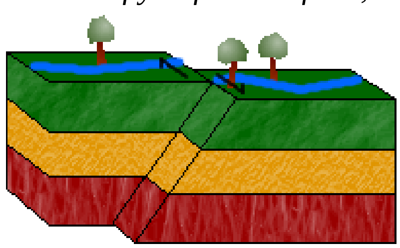
Draw a sketch showing the effects of wave dispersion on the propagation velocity of surface waves for the following soil profiles:
Answer:



What is the propagation velocity of P-waves in Water, and why is this value of importance:
Answer: It is about 1500 m/s and it is important because when this P-wave velocity is measured, recorded, it might indicate presence of water underground.

Why is significant seismic energy observed to propagate for larger distances in the Eastern US.
Answer: Rock formations in the Eastern US are stiffer than those in California, allowing seismic waves to travel faster and farther, with little attenuation.

Draw a sketch of right lateral fault system.
Answer: If you face the fault, the other side moves to your right.



Near the Fault and far away different buildings suffer. Briefly explain.
Answer: Stiffer buildings (with high natural frequency short natural period) may suffer close to the fault where high frequency waves might dominate. Farther away, wave dispersion causes energy to gradually shift towards lower frequencies causing potential damage to flexible structures with longer natural periods (lower natural frequencies).

Why do most earthquakes occur in narrow localized bands within the globe?
Answer: These bands are the locations of interaction between plate boundaries (based on the plate tectonic theory).

How much more energy is released by a magnitude 6 earthquake compared to a magnitude 5?

Answer: Approximately 30 times

Why are seismologists expecting an earthquake north of Los Angeles anytime now?
Answer; Because this is the location of the segment of the San Adreas Fault that has not slipped in recent times, and is due to slip based on historical records.

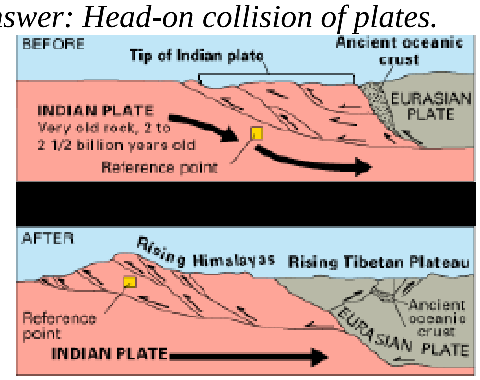
Mention some limitations of Richter Local Magnitude (M_L).
Answer: M_L is based only on the peak acceleration of the shear-wave, and consequently does not account for the duration of shaking. Also, peak acceleration of the shear wave does not increase in proportion to released energy particularly for large Magnitude earthquakes.

Mention one limitation of P-wave magnitude M_b.
Answer: Mainly good for deep focus earthquakes (i.e., not so good for California Shallow focus earthquakes)

What is a thrust fault.
Answer: Thrust faults typically have low dip angles. A high-angle thrust fault is called a reverse fault.

What type of earthquake is associated with volcanic activity? Draw a sketch.
Answer: Subduction-type earthquakes

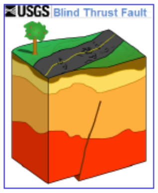
e) What type of faulting mechanism is associated with the Himalayan Mountains? Draw a sketch.
Answer: Head-on collision of plates.



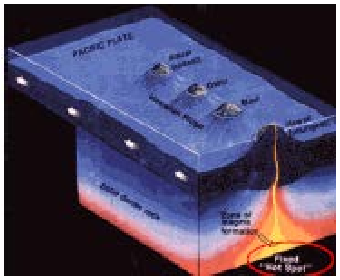
- What percentage of the earthquakes is proven to come from the ocean ridge system? And what percentage of the seismic energy? Answer: *The ocean ridge system is responsible for 10 % of the earthquakes and 5% of the seismic energy.*

From what kind of system is most seismic energy coming from?
Answer: From the (boundaries between tectonic plates

Draw a sketch of a blind fault
Answer:

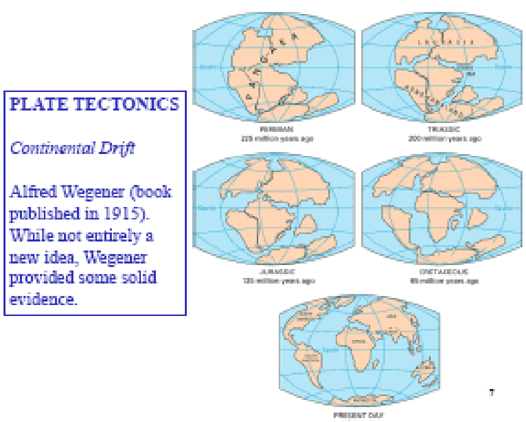


Why are the Hawaiian Islands in a chain?
Answer:



What is the age of Planet Earth
Answer: About 4.5 Billion years

What is the theory of Plate Tectonics.
Answer:



– Describe the Halocene and Plastocene formations?
Answer: Halocene formation – last 10000 years; Plastocene formation – between the last 10000 years and 2 and a half million years.

From what period do the older rocks come from?
Answer: The Cambrian and the Precambrian.

– What motion is characterized by "Near Source"?

Which type of wave arrives first, and is later followed by…… Draw a sketch, and show how we can guess proximity to source from shape of record.

Answer:

Why is it assumed that one of the deep zones within the earth is in a fluid state ?
Answer: No Shear waves are observed to travel through it

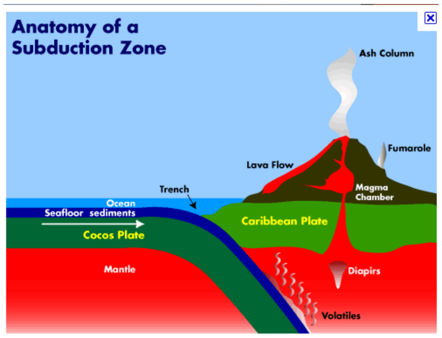
Mercalli Intensity is affected by local conditions (give 2 examples).
Answer: Local type of construction, and local site conditions

For earthquake engineering, what has been the problem in the Marina District of San Francisco and why?
Answer: Soil Liquefaction (saturated un-compacted sediments are vulnerable to liquefaction)

Question:
a) Released energy from an earthquake depends on (choose one only):
1) Ruptured surface area along the fault plane
2) Amount of movement or slip along the fault plane
3) Both
b) Which Earthquake Magnitude scale employs the concept(s) above.
Answer: Moment Magnitude

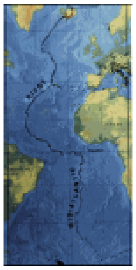
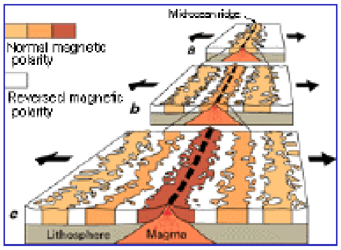
What is the Volcanic activity in Washing ton State a consequence of (be specific please).
Answer: Nearby subduction-type earthquakes (oceanic crust -continental crust)

Draw a sketch of a subduction type fault mechanism.
Answer:



Draw a sketch of a spreading ridge. How do we know that such ridges have been spreading for a long geological time?
Answer:

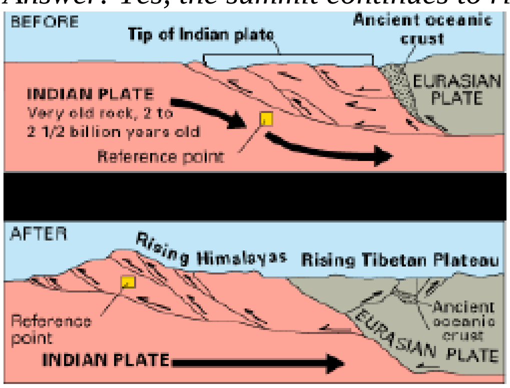
http://pubs.usgs.gov/gip/dynamic/understanding.html

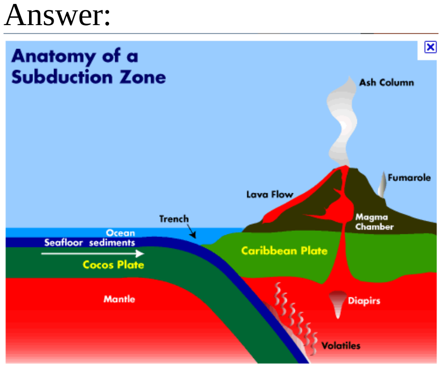
The Mid-Atlantic Ridge, which splits nearly the entire Atlantic Ocean north to south, is probably the best-known and most-studied example of a divergent-plate boundary. (Illustration adapted from the map This Dynamic Planet http://mineralsciences.si.edu/tdpmap/ *.)* 13

What are the Himalayan Mountains a consequence of (Draw a simple sketch)? Is the summit elevation expected to continue rising?

Answer: Yes, the summit continues to rise



Volcanic activity is strong in the State of Washington, but much less so in California. Why?
Answer:



What is a typical range of earthquake focal depth in California.? What are such earthquakes classified as?

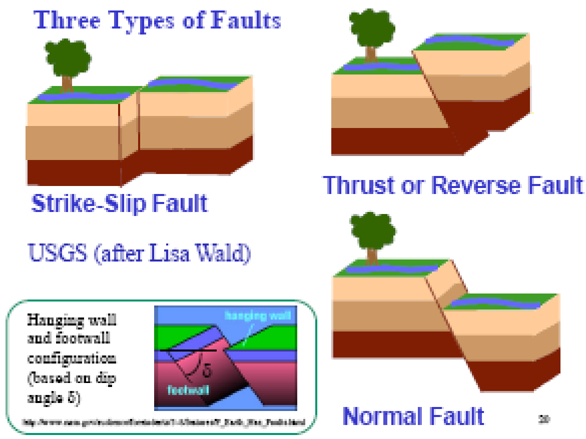
Answer: Shallow earthquakes, with focus of in the vicinity of 15 miles or less below ground (generally)

Magnitude might not be adequately descriptive of energy levels affecting the built environment. Why?

Answer: Because the earthquake focus might be deep and little of the released energy would reach ground surface.

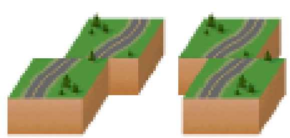
Sketch three types of faults (and show Hanging and Foot walls)

Answer:



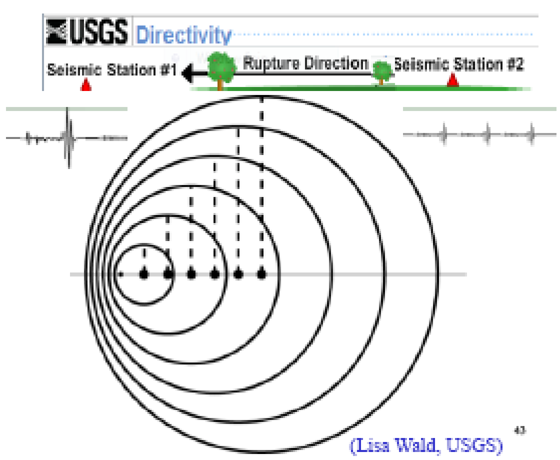
Show Right Lateral and a Left Lateral strike slip fault configurations

Answer:



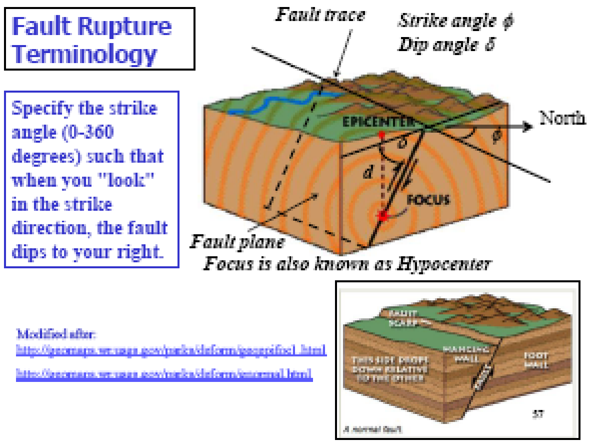
right-lateral and left-lateral

Discuss the figure below as relates to Directivity

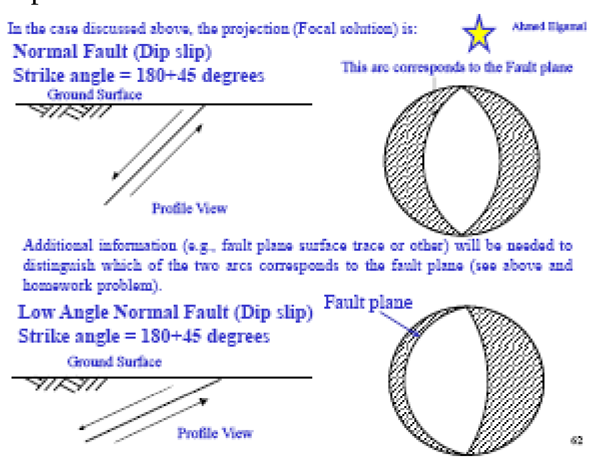


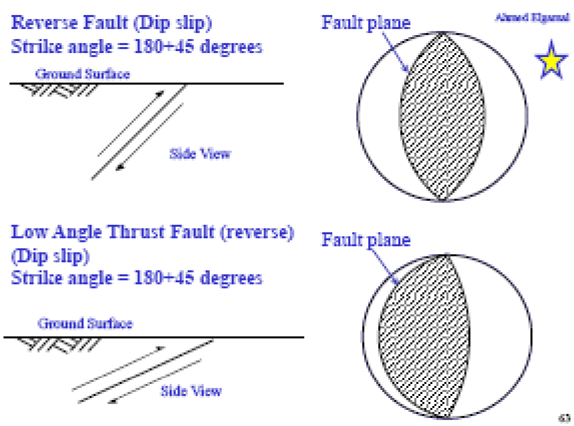
Draw an annotated sketch showing fault rupture terminology

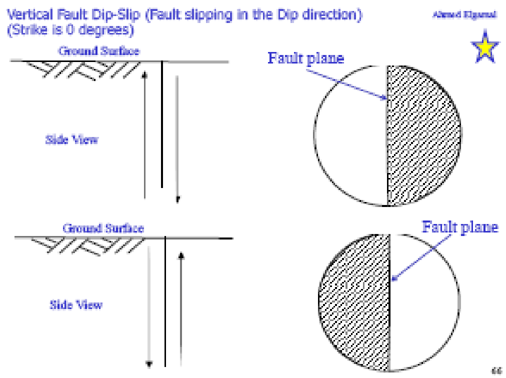
Answer:



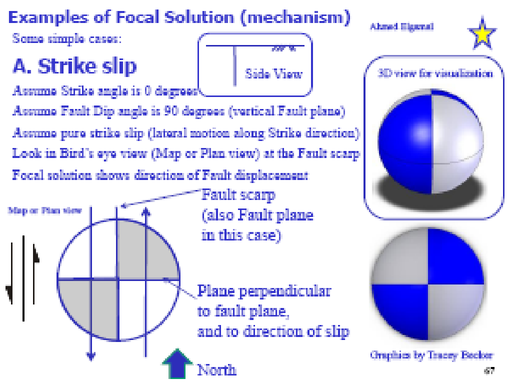
Important Focal Mechanisms



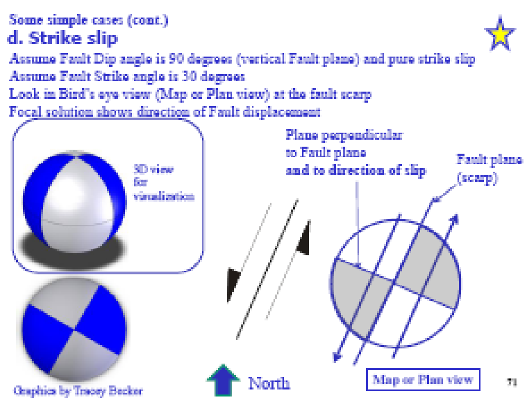




Vertical Fault Dip-Slip (Fault slipping in the Dip direction)
(Strike is 0 degrees)
Ahmed Elgamal
Ground Surface
Side View
Fault plane
Ground Surface
Side View
Fault plane
66



Examples of Focal Solution (mechanism)
Ahmed Elgamal
Some simple cases:
A. Strike slip
Side View
Assume Strike angle is 0 degrees
Assume Fault Dip angle is 90 degrees (vertical Fault plane)
Assume pure strike slip (lateral motion along Strike direction)
Look in Bird's eye view (Map or Plan view) at the Fault scarp
Focal solution shows direction of Fault displacement
3D view for visualization
Map or Plan view
Fault scarp
(also Fault plane
in this case)
Plane perpendicular
to fault plane,
and to direction of slip
North
Graphics by Tracey Becker
67



Some simple cases (cont.)
d. Strike slip
Assume Fault Dip angle is 90 degrees (vertical Fault plane) and pure strike slip
Assume Fault Strike angle is 30 degrees
Look in Bird's eye view (Map or Plan view) at the fault scarp
Focal solution shows direction of Fault displacement
3D view
for
visualization
Plane perpendicular
to Fault plane
and to direction of slip
Fault plane
(scarp)
Graphics by Tracey Becker
North
Map or Plan view
71

Intro to Equake Engineering

Briefly explain why unreinforced masonry performs poorly during an earthquake.
Answer: Solid brick masonry is very heavy and its tensile strength is low, and therefore its flexural strength per unit weight for in-plane and out-of-place seismic forces, is very small.

List three components of a bridge that are susceptible to being damaged during an earthquake.
Answer: In the Superstructure, Column, Expansion Joints, Abutment…

Where might one expect to find examples of lateral spreading caused by the Kobe earthquake (1995)?
Answer: Around port facilities

List two natural disasters that can be triggered by an earthquake.
Answer: Tsunami, Landslide

What type of construction was particularly damaged during the 1933 Long Beach Earthquake?
Answer: Unreinforced masonry

Which earthquake demonstrated that earthquakes pose a risk to eastern U.S. cities?
Answer: New Madrid earthquakes (1811-1812)

Name or describe an unexpected problem associated with steel structures during the Northridge 1994 earthquake.
Answer: Weld failure; Crack initiations at the weld of beam-column connections and propagating through the column

Describe an example of a problem associated with a surface rupture during an earthquake.
Answer: Situations associated with the permanent offset such as breakage of underground pipelines .

List the most notable item to come out of the 1940 Imperial Valley (El Centro) earthquake.
Answer: The El Centro Acceleration record.

Describe a notable feature of the 1985 Michoacan earthquake and the damage it caused.
Answer: The earthquake occurred off the coast, but the heaviest damage was in taller buildings located in Mexico City. This was due to the frequency of the buildings and the soil they were

built on matching the low frequency waves which induced resonance in the soil and buildings. This combination of events caused tremendous damage.

True or False.
The Northridge earthquake occurred on the San Andreas fault.
Answer: False – it was on a blind fault.

This earthquake, which was the largest earthquake ever recorded (at the time) and caused a series of seismic sea waves (tsunami) which also caused numerous casualties and extensive property damage in Hawaii and Japan, occurred in which country?
Answer: Chile

This failure mechanism was a major contributor to the widespread collapse of buildings during the 1999 Duzce (Turkey) earthquake.
Answer: Pancake-type collapse

Fill in the Blank:
A flexible low natural frequency structure may suffer more damage if ___***(far from)***_____ the earthquake source

What is the "Mexico City" earthquake mechanism in terms of frequency, number of cycles, incoming motion amplitude, building characteristics and resulting response.
Answer: very low amplitude seismic waves arrive to Mexico city from the west coast of Mexico approximately 400 km away. The shaking lasts for a long time, but the amplitudes of input excitation are very small. Because part of mexico city is built on deep sediments of soft (but linear) clay, in a Bowl-like Basin, the clay moves back and forth like a Bowl of jelly. It gradually builds up higher and higher amplitudes at ground surface due to resonance of the deep soil layer.Buildings on top of this soil, whose natural frequency matches the soft soil layer natural frequcny, end up experiencing harmonic motion at their base, in resonance with their own natural frequency, which is a very bad scenario that caused many building collapses.

When soil liquefies, some buried structures may float upwards. Why?
Answer: They are lighter than the surrounding liquefied soil and the virtually zero soil strength during liquefaction allows them to float upwards.

Ground Motion Parameters

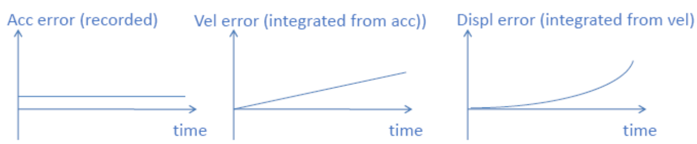
Show the drift errors that are associated with integration of an earthquake acceleration record to derive velocity and displacement time histories.

Answer:

As such (particularly for older records, the shown velocity and displacement records are not the direct outcome of time integration of the recorded acceleration record. Mainly, offset and drift in these records (with time) have been removed.

Note: When a strong motion instrument "triggers" to record (save) an earthquake motion, the prior five seconds of excitation are also stored as part of the record. This helps ensure tat the recording starts from a near zero value.

Nevertheless, any small error can be greatly magnified when integration is performed to calculate the corresponding displacement and velocity time histories.

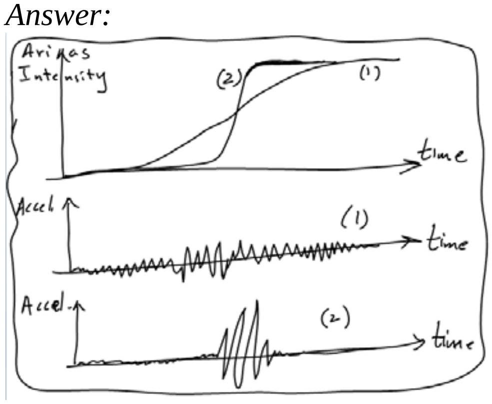


Define PGA, PGV, PHA, PVA

Answer: PGA =Peak Ground Acceleration, PGV = Peak Ground Velocity, PHA =Peak Horizontal Acceleration, PVA =Peal Vertical Acceleration

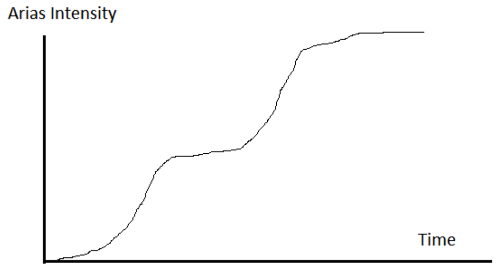
Draw a sketch of the earthquake time histories that correspond to the following Arias Intensity figure

Answer:

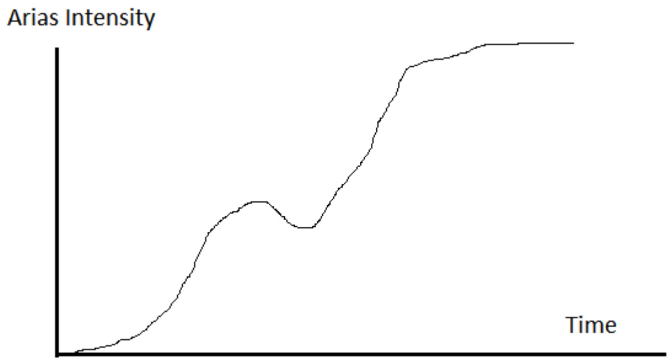


Draw a sketch of an Arias Intensity versus time figure, where the earthquake time history depicts two time intervals of strong shaking, with a short duration of mild shaking in between

Answer:



Can the figure below be representative of Arias intensity versus time



Answer: No, because it shows an interval of decrease in values, which cannot be true (since Arias Intensity is an integration of acceleration squared values (a^2), leading to continued increase or sustained values

In the Arias Intensity expression, $I_a = \frac{\pi}{2g}\int_0^\infty \left[a(t)^2\right]dt$

(a) Why is the term “Arias” used?
Answer: This Intensity scale was proposed by Professor Arturo Arias

(b) Why is the term “Intensity” used?
Answer: Because it represents a summation of the vibrational effects over the duration of the shaking event (due to the integration process)

For an attenuation relationship that predicts peak ground acceleration (PGA) as a function of distance from fault or epicenter, fill in the blanks below:

PGA = *function of* (“ ” , “ ”, “ ”)

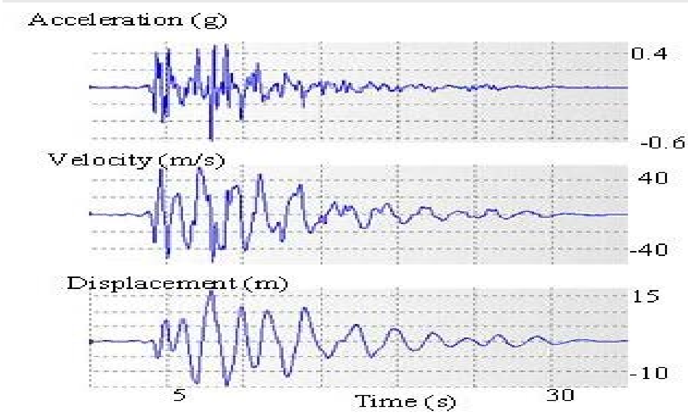
“ ”

“ ”

“ ”

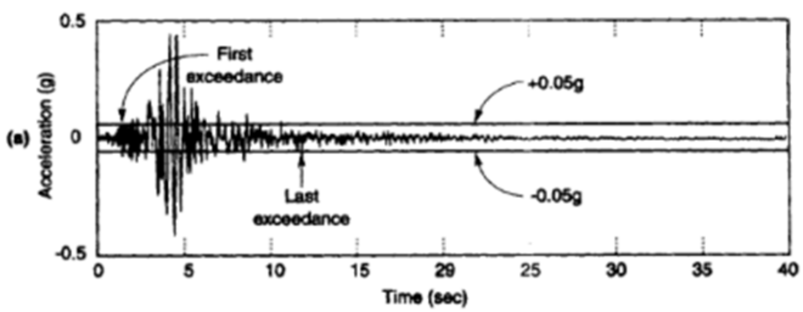
Mark the figures below as Acceleration, Velocity, and Displacement

Answer: Acceleration shows high frequency oscillation, and displacement is smoothest



What is Bracketed Duration, and why is it possibly of importance

Answer: Bracketed duration is the measure of the time between the first and last exceedance of a threshold acceleration (e.g., 0.05 g). It is useful for scenarios where repeated cyclic loading can be detrimental (e.g, Liquefaction). In the figure below (for a 0.05g threshold, it is about 10 seconds).



What is an attenuation relationship for ground motion parameters

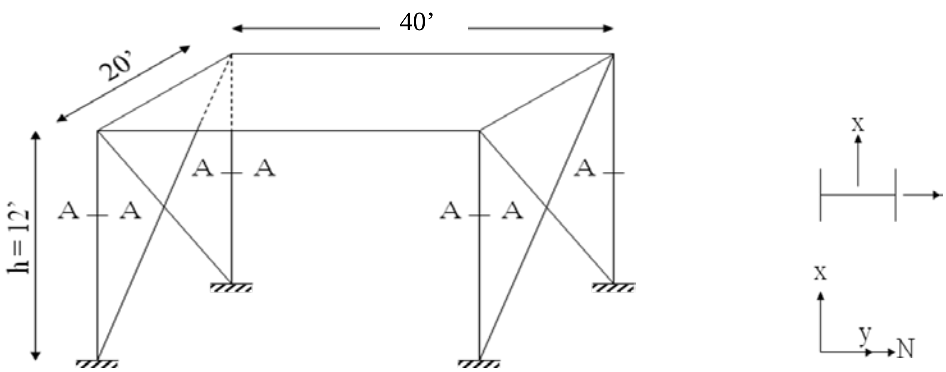
Answer: An example is presented below for PHA (attenuation o Peak Horizontal acceleration with distance), showing dependence on M (Local Magnitude or Surface wave Magnitude > 6) and R (closest distance from fault in km < 50 kms).

$$\ln PHA(g) = -4.141 + 0.868M - 1.09\ln[R + 0.606\exp(0.7M)]$$

Bracing

- Find the natural frequency ω in the braced X-direction (with cable-type bracing). **Make sure to include the contribution of columns to the stiffness in this direction (small as it may be).**

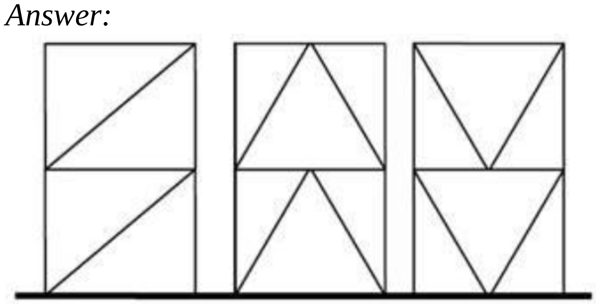
Note that: $m = \dfrac{w}{g}$



Given:
Weight of roof = 40 lb/ft^2
k_{brace} = 60 kips/in
$k_{col(x\text{-}direction)}$ = 2 kips/in
Assume g = 400 in/sec^2 (approx. for the purpose of rough hand calc.)

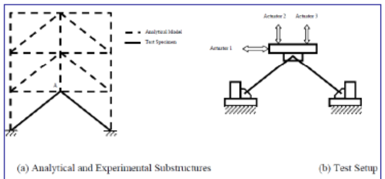
- For the braced frame above, when subjected to the El Centro earthquake motion, find the peak base shear force.

Name three types of Bracing systems
Answer:



Bracing Systems (a) Diagonal, b) Chevron, and c) V-Braced

Why does the Zipper Frame improve overall performance? Draw a sketch
Answer: It provides much ductility to the overall system



(a) Analytical and Experimental Substructures (b) Test Setup

Figure 6 – Fast Hybrid Tests at the University of Colorado

Behavior of Braced Steel Frames with Innovative Bracing Schemes

A NEES Collaboratory Project

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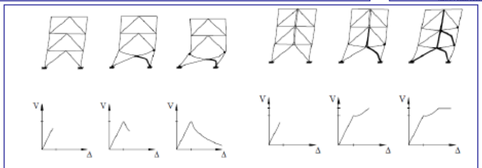

(a) Conventional braced frame (b) Zipper frame

Figure 2 – Comparison of the collapse mechanism and load-displacement relationships for zipper and conventional braced frames (Khatib et al. 1988).

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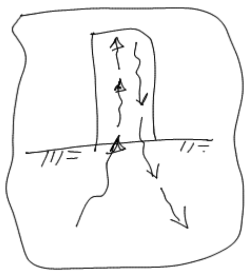
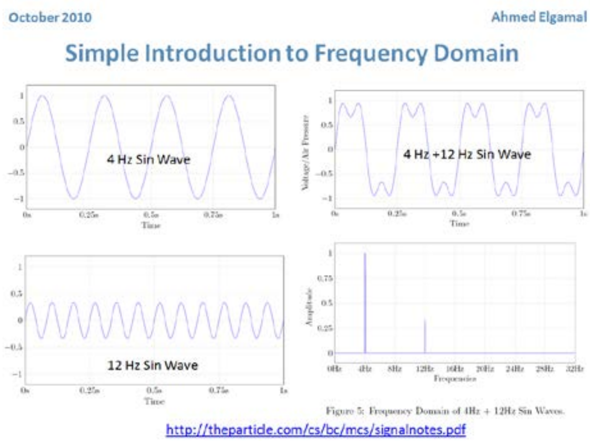
Question: Draw a sketch of the a Buckling restrained Brace

Miscellaneous

– Draw a sketch showing the concept of radiation damping.
Answer:

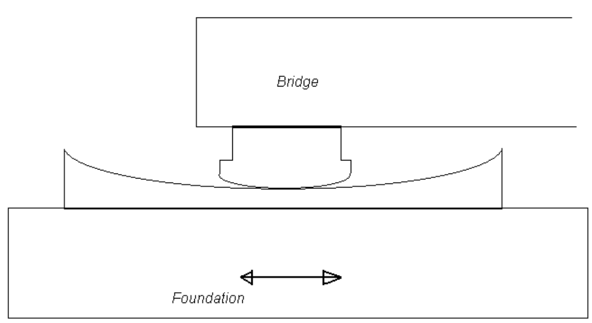


Can any earthquake record be represented by a summation of sines and cosines (draw a sketch)?
Answer: Yes



Can any earthquake by represented by sines and cosines?
Answer: Yes, any signal can be represented by a sum of different functions. (Fourier sines and cosines)

-A seismic isolator under a bridge support was designed to be of the shape:



For earthquake resistance, mention two benefits of the above curved shape.
Answer: Lateral motion is somewhat mitigated by having to lift the structure upwards, and the inclined surface helps to bring the structure back to its original position after the earthquake.