Optional Homework

1) Find mode shapes and natural frequencies of a cantilever shear beam defined by:

$$\rho \ddot{u} = Gu'' \tag{1}$$

And boundary conditions:

$$u(0) = 0 \text{.....} Gu'(h) = 0 \tag{2}$$

Where ρ is mass density, G is shear modulus, and h is beam height.

Hint: Note that the shear wave velocity V_s is defined by:

$$V_s = \sqrt{G/\rho} \tag{3}$$

and the natural frequencies in Hertz are:

$$f_n = (2n-1)V_s/4h \text{.....} n = 1,2,3,.. \tag{4}$$

2) Use a finite element program (bending beam elements) to model a 30 m cantilever bending beam ((choose EI to represent a building), and subject it to a unit load at the top. Compare the result from 3 different meshes of 2 elements, 6 elements and 10 elements. Repeat specifying a zero rotation at the top. Compare the results to the theoretical deflection at the beam top (both cases, free at the top, and zero rotation at the top).

3) Use a Finite Element program (Beam Elements) to model a cantilever bending beam 30 m in height (choose EI and m to represent a building). Build 3 meshes, of 2 elements, 6 elements and 10 elements, and compute mode shapes and natural frequencies in each case (focus on first 4 mode shapes and frequencies). Plot the mode shapes in each case. Compare the resonant

frequency results and verify based on the equations for natural frequency provided in this handout.