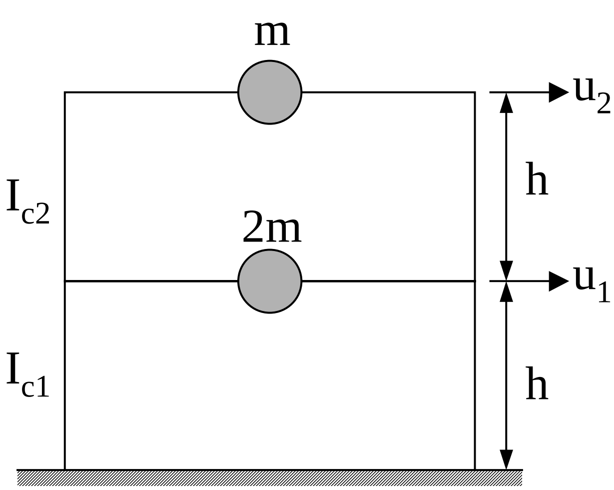
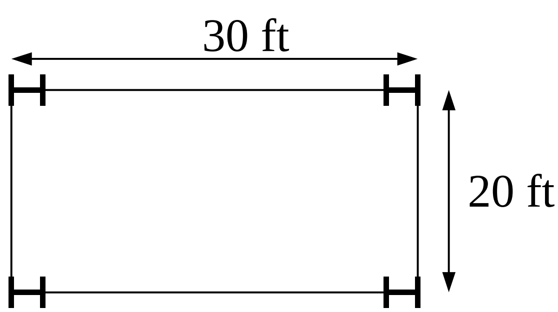
1)

(a) For the two story building shown below, define the two-degree-of-freedom free vibration matrix equation in terms of k and m. Using this matrix equation, determine the natural frequencies ω_1 and ω_2 (in terms of k and m). Using these expressions for ω_1 and ω_2, plug in numerical values and determine ω_1 and ω_2 in radians. For each natural frequency, define and sketch the corresponding mode shape.



Elevation View



Plan View

Use:

E_{steel} = 29,000 ksi
I_{c1} = 164.8 in^4
I_{c2} = 82.4 in^4
h = 12 ft

Dead weight (lumped at floor level)

$$m = \frac{w}{g} = (30\text{lb/ft}^2)\frac{(30\text{ft}\times 20\text{ft})}{386\text{in/sec}^2} = 46.63\text{lb}\cdot\text{sec}^2/\text{in} = 0.04663\text{kip}\cdot\text{sec}^2/\text{in}$$

$$k_{column} = \left(\frac{12EI_c}{h^3}\right) \Rightarrow k_{floor} = 4\left(12\frac{EI_c}{h^3}\right) = \frac{48EI_c}{h^3}$$

(b) Verify that the modes are orthogonal as expected.

(c) Normalize the first mode such that $\phi_1^{\mathbf{T}}\mathbf{m}\phi_1 = 1.0$

(d) Use the normalized first mode (from above) to verify that $\phi_1^{\mathbf{T}}\mathbf{k}\phi_1 = \omega_1^2$

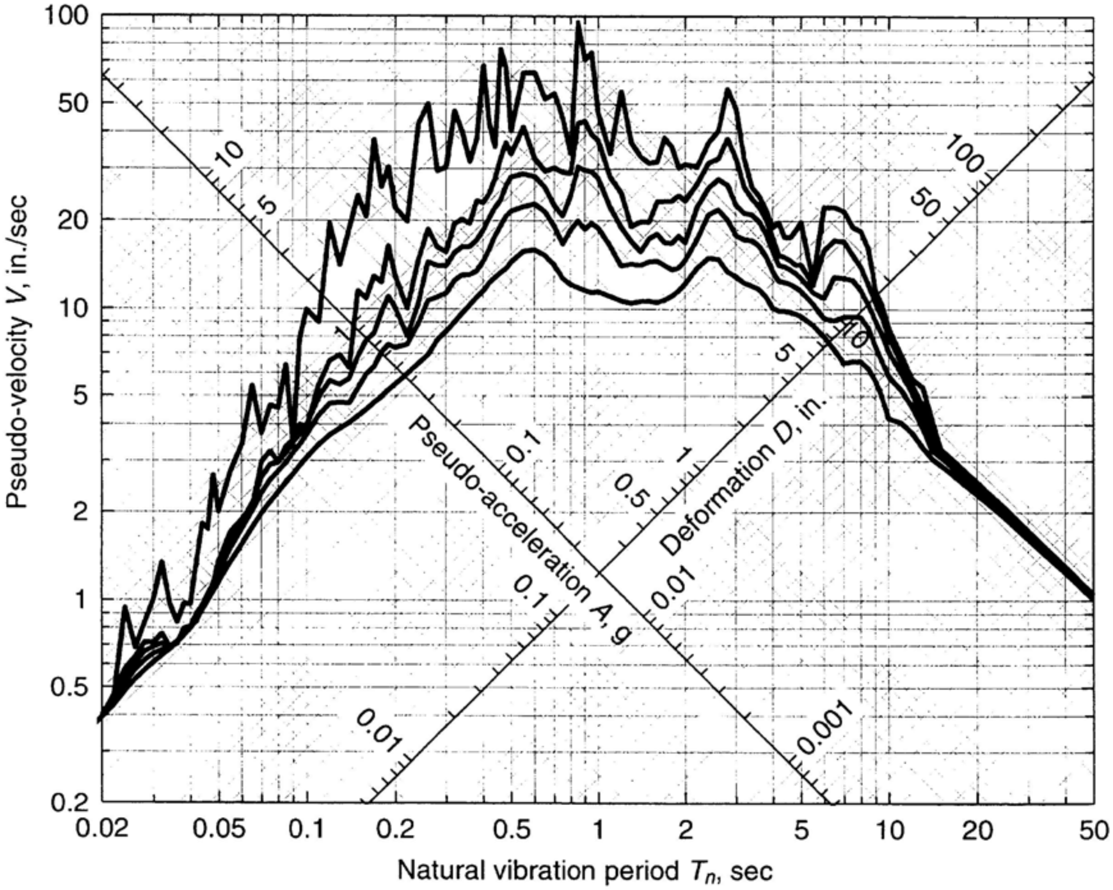
(e) Use the El Centro Response Spectrum and a damping ratio of 5% to estimate the maximum base shear and moment.

(f) Find a_0 and a_1 in $\mathbf{c} = a_0\mathbf{m} + a_1\mathbf{k}$ for a viscous damping of 5% in modes 1 and 2.

(g) Using Newmark's Method and Modal Analysis (following the procedure outlined in *Step-by-Step Procedure for Using Newmark's Method & Modal Analysis*), solve for the response of the 2DOF model subjected to the El Centro time history. Include plots of relative displacement and absolute acceleration for each floor.

(h) Compare peak relative displacements and absolute accelerations found in Part (g) with those from the response spectrum.

Response spectrum for El Centro ground motion ζ = 0, 2, 5, 10, and 20%.



Hints for Problem 1:

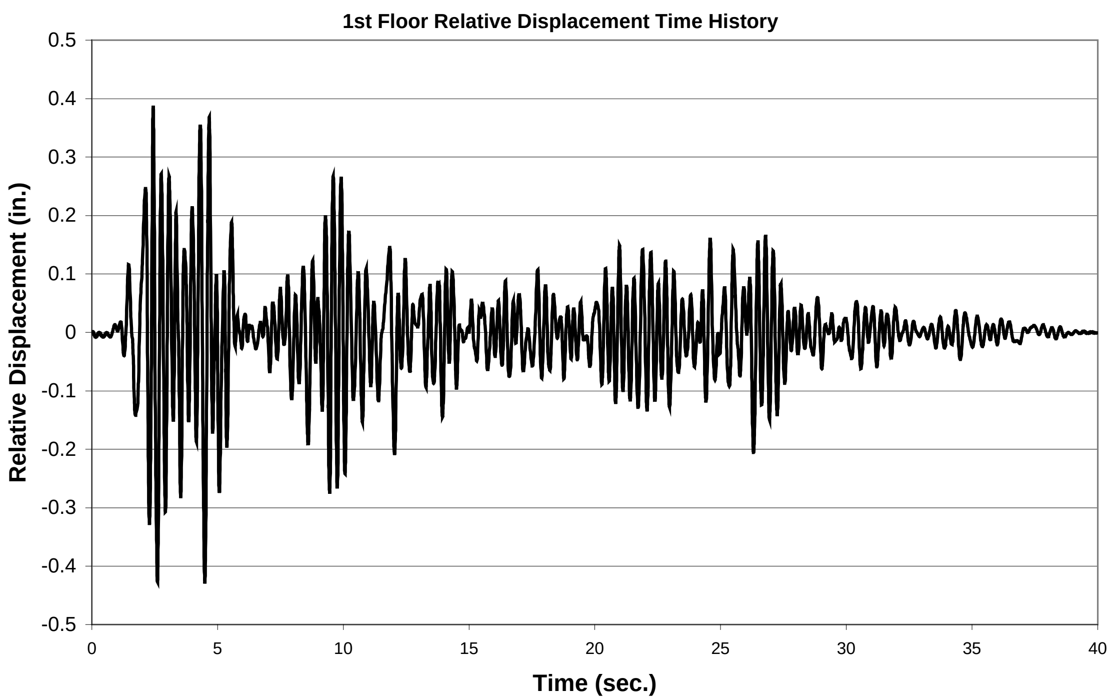
When solving this problem, you should find in part (a)

ω_1 = 20.295 rad/sec ,

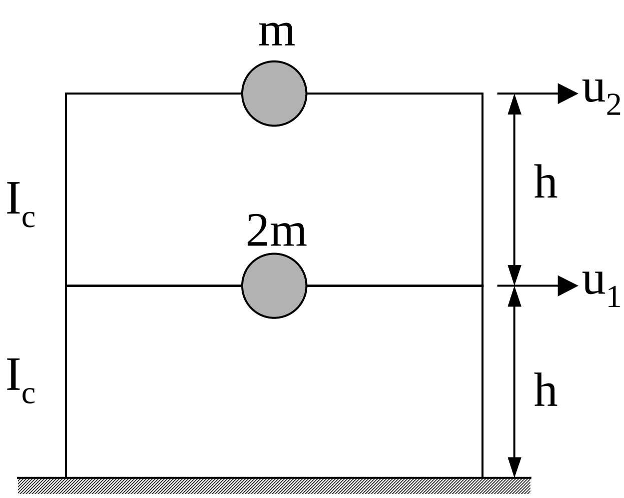
$$\Phi_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix},$$

In part (e), $V_{0\,\max} \approx 38.67\ kips$.

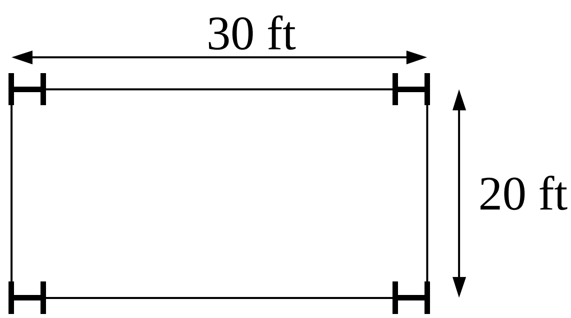
In part (g), you plot for the 1st floor relative displacement should look like the one below.



2) (Optional) Repeat problem (1) for the two-story building shown below.



Elevation View



Plan View

I_c = 82.4 in^4, and all other data same as above. Which structure (Problem 1 or Problem 2) has lower resonant frequencies? Why ?